1963

A structural analysis and linear programming within an input-output model emphasizing agriculture in India

Marudarajan Ramachandran
Iowa State University

RAMACHANDRAN, Marudarajan, 1926–
A STRUCTURAL ANALYSIS AND LINEAR PROGRAMMING WITHIN AN INPUT-OUTPUT MODEL EMPHASIZING AGRICULTURE IN INDIA.

Iowa State University of Science and Technology
Ph.D., 1963
Economics, agricultural

University Microfilms, Inc., Ann Arbor, Michigan

A STRUCTURAL ANALYSIS AND LINEAR PROGRAMMING WITHIN AN INPUT-OUTPUT MODEL EMPHASIZING AGRICULTURE IN INDIA

by

Marudarajan Ramachandran

A Dissertation Submitted to the
Graduate Faculty in Partial Fulfillment of
The Requirements for the Degree of
DOCTOR OF PHILOSOPHY

Major Subject: Agricultural Economics

Approved:

Signature was redacted for privacy.

In Charge of Major Work

Signature was redacted for privacy.

Head of Major Department

Signature was redacted for privacy.

Dean of Graduate College

Iowa State University
Of Science and Technology
Ames, Iowa

1963

TABLE OF CONTENTS

	Page
INTRODUCTION	1
Nature of Input-output Analysis	1
Historical Background	2
Objectives of Study	6
PARTIAL ANALYSIS, INPUT-OUTPUT AND GENERAL EQUILIBRIUM	8
Partial Analysis and the Input-output Model	8
General Equilibrium and the Leontief System	10
AGGREGATE GROWTH MODELS AND THE INPUT-OUTPUT SYSTEM	17
The Harrod-Domar Model	18
The Mahalanobis' Model	21
Macro Economic Models and the Dynamic Input-output System	35
A Short Term Planning Model for India	38
A Long Term Planning Model	47
THE LEONTIEF OR INPUT-OUTPUT SYSTEM	57
Mathematical Model	58
A Numerical Example	67
Assumptions of the Input-output Model	75
AGGREGATION AND SECTORAL CLASSIFICATION	77
Problem of Aggregation	77
Conceptual Background for the Indian Inter-industry Table	84
Sources and Estimation Procedures	90

	Page
EMPIRICAL APPLICATIONS	93
Structural Analysis	93
Programming Applications	113
A Linear Programming Model within the Input-output Framework	130
A Variable Capital Programming Model	148
SUMMARY	181
ACKNOWLEDGMENTS	185
LITERATURE CITED	187
APPENDIX A	195
APPENDIX B	198
APPENDIX C	201

INTRODUCTION

In the recent past, most countries in Asia, Europe and Latin America have been formulating and implementing development plans. During the process of planning, there is the need for appraising individual investment projects. Often all investment projects are interrelated. Unless the detailed interindustrial relations are known, the input-output relations among industries can hardly be grasped. For example, an investment in one industry requires the input of raw materials from many other industries, whose production must be increased; hence more investment must be made in these industries. This endless process of considering all the interrelations is the theme of input-output analysis or interindustrial analysis.

Nature of Input-output Analysis

Input-output analysis provides a compact and systematic arrangement of different sectors or industries of an economy. It allows the goods and services associated with each industry to be identified as sales and purchases, depending on whether they are outputs or inputs, respectively. These quantitative relationships are utilized to investigate the various facets of the economy. Interindustry techniques have been applied for both structural analysis and for orienting government policies. Though most of the research in the field has been designed to reveal the quantitative significance of various types of interdependence of the economy, quite a few countries have undertaken the task of building interindustry tables so as to serve as an aid to planning and economic development. Beyond revealing the network of interrelationships, the interindustry techniques have so far been of limited value for prediction of future events.

Historical Background

Although intersectoral relationships in economics can be traced back to the days of Tableau Economique of Quesnay (1758), the modern theoretical form for the interdependencies was provided by Leon Walras (1874) only in the late nineteenth century. Empirical explorations in the interindustry analysis began with the studies of Professor Leontief (1951) on the American economy published in 1936 and 1941. His first work was the 1919-29 study of American economy, which was followed by a table for 1939 constructed by members of the Harvard Economic Research project (1953). Since then, extensive work has been undertaken and the United States Department of Labor (1952) has prepared an elaborate table of two hundred sectors for 1947, in which about four hundred and fifty industries have been detailed out and distinguished. Cornfield, Evans and Hoffenberg (1947) investigated the industrial implications of the attainment of full employment in 1950 using the 38 x 38 input-output matrix based on the 1939 table prepared by Leontief (1951). Barnett (1954) evaluated the results of the study by using multiple regression techniques and concluded that the input-output projections were the best for all industries. However, the study also indicated that alternate techniques were better, when individual industry groups are considered. The studies initiated by the U. S. Bureau of Labor Statistics, the Bureau of Mines, the Air Force and the Harvard University yielded detailed indices of price and production from the base year (1947), capital coefficients by separate types of equipment for each industry, inventory requirements and input structures for certain strategic military industries. An extension of the input-output technique for solving practical problems was the "emergency model" built up to explore the possibilities of post-Korean defense build up.

In the United Kingdom, Barna (1956) applied input-output analysis to ascertain the import content of different elements of final demand and to study the effects of wage adjustment on prices. Detailed studies have also been done in Japan by way of structural analysis of the direct and indirect requirements of final demand elements for imports for labor and capital. The Economic Commission for Latin America (1956, 1957, 1958) concentrated on the construction of separate import matrix for several Latin American countries so as to allow for a detailed analysis of import requirements.

Several regional input-output models have been developed by decomposing the aggregate production and consumption by regions. The regional models can broadly be classified as (i) international trade models, (ii) interregional input-output models and (iii) the interregional programming models. Neisser-Modigliani (1953) constructed a world trade model for three commodities and six regions, with exports and imports as endogenous variables and national income, industrial production and prices as exogenous variables. The input-output model of Moses (1955) for United States economy and the model of Chenery (1953) for Italy are more detailed than other models in respect to the number of commodities, but the number of regions are less, three in the former and two in the latter. Both the models treat production levels and inter-regional shipments as endogenous variables, while final demand is considered exogenous. The programming analysis of Hender-son (1958) for the United States coal industry determines production levels, but considers consumption levels given in each region. On the other hand, in the model of Fox (1953) for livestock feed, demand in each region is dependent on price, while the production is assumed as given. The optimum supply pattern is then determined. Isard (1951, 1953) and Leontief (1953) proposed a "balanced regional model", which could be used to disaggregate the national aggregate.

Peterson (1953) and Peterson and Heady (1955) constructed a five sector model of the U. S. economy for 1949, 1939 and 1929, laying special emphasis on agriculture. Schnittker (1956) and Schnittker and Heady (1958) pursued the work and constructed a regional model for agriculture for 1949. In this model, there are six regions, subdivided into primary and secondary agriculture and six aggregate national industry sectors. Carter (1958) and Carter and Heady (1959) developed a regional model for United States, with ten regions and nine commodity groups and studied economic interrelationships among agricultural regions and between agriculture and rest of the economy. Regional studies have also been carried out by Derwa (1957) for Belgium, by Bauchet (1955) for France, by Artle (1959) for Sweden, by the Kansai Economic Federation (1958) for Japan and by the Municipal Bureau of Statistics (1953-54) for Holland.

The last decade has witnessed a rapid and energetic growth in the input-output studies in many parts of the world. Chenery and Clark (1959) in summarizing the present research in the field observe that the applications that have been made so far constitute illustrations of its usefulness having varying degrees of realism; but, however, they consider that the results must still be regarded experimental. The nearest approaches to policy and programming uses of the input-output technique have been attempted only in a few countries, as United States and Italy. Applications to government planning and policy orientation have been made on a limited scale in a number of countries, as the Netherlands, Israel, Argentina, Japan, Norway, Yugoslovia, Peru, India and a few others.

Objectives of Study

The general objective is to investigate problems associated with the applications of the input-output model. Specific objectives are:

1. To appraise the similarities and dissimilarities between the general equilibrium analysis of the classical and neo-classical schools and the input-output analysis.
2. To trace the relation between aggregate growth models and input-output framework and to review some of the applications of input-output technique for short term and long term planning in India.
3. To discuss the problems inherent in aggregation and consider the conceptual framework for the input-output table of the Indian economy.
4. To use the presently available Indian table in an aggregated form for making an empirical application of the input-output technique.
5. To investigate the interdependencies between the different sectors of the Indian economy and systematically

analyse the destination of flows of goods and services and the purposes for which they are absorbed.

6. To formulate a programming model within the input-output framework to allocate the given capital resources in an optimal manner.

7. To develop a variable capital programming model using the input-output coefficients and to obtain the optimum levels of output at different discrete levels of capital availability.

PARTIAL ANALYSIS, INPUT-OUTPUT AND GENERAL EQUILIBRIUM

In this section, we consider the approaches of partial analysis and the input-output system and discuss the similarities and dissimilarities of the Walrasian general equilibrium and the Leontief system.

Partial Analysis and the Input-output Model

The partial equilibrium analysis, propounded by Marshall (1890) and extended by his followers, explains the reactions of producers and consumers of a given commodity to each other's behavior and thereby determines price and output levels in a given market under *ceteris paribus* conditions. The relations between the particular industry supplying the commodity and the consumers are expressed by a set of supply and demand functions. Changes in the output levels or income of the household alter the demand functions, while changes in the uses for inputs affect the supply functions of the industry.

The Leontief input-output system, on the other hand, is an aggregate system, which assumes fixed input proportions. Supply and demand in each market is brought about, not by changes in price, but by changes in output. In view of the assumption of fixed input proportions, changes in the production of any one sector affects all the other sectors of the economy. The profit maximization, which is the cornerstone of partial analysis, is not explicit in the Leontief analysis. Besides, partial analysis generally concentrates only on one or a few of the sectors of an economy and it is further assumed that other sectors are not affected by the changes in the sector producing the commodity in question, which is far from reality. Professor Leontief (1951) has stressed this aspect in defending his system as follows:

> The principal merit of the general equilibrium theory is that it enables us to take account of the highly complex network of interrelationships which transmits the impulses of any local primary change into the remotest corners of the economic system. While in the case of partial analysis, which operates simultaneously with only two or three variables, the interrelationships among these few elements can be perceived directly, such intuitive inference becomes practically impossible as soon as the number of variables increases up to four or five, not to say ten or twenty.

Regression analysis and input-output

In the regression analysis, usually a small number of independent variables are used to estimate a dependent variable. In contrast, in the input-output analysis there are a large number of independent variables. Even by inclusion of larger numbers of variables in the regression analysis, one is not sure that the results would really improve. The input-output models seem to have one advantage over the regression models in that any discrepency can be checked by point by point comparisons to locate the error in the structure or in the demand. In the case of regression analysis, there is no scope for tracing backwards and one has to simply give up, if the results are not up to theoretical expectations. The input-output analysis attempts to establish causal sequences and errors in estimates are not considered to be the result of stochastic processes, but due to the failure to identify correctly the parameters of demand or structure. As such, discrepencies could always be reconciled in an input-output model and there is no scope for hindsight in a regression model.

General Equilibrium and the Leontief System

The interdependencies of economic variables in the system were considered by Quesnay, as early as 1759. Political arithmeticians as Petty, King and others recognized in the structure of their accounts the intermediate and final product relationships even earlier. Walras (1874), Pareto (1911) and Barone (1938) described interdependencies of the economy in a system of mathematical equations, but the system was not applied for actual problems. The Walrasian system determines prices and quantities of services and goods supplied and goods demanded under equilibrium conditions, when coefficients of production, utility functions and supply functions are assumed to be known. The input-output systen, on the other hand, does not allow for changes in relative prices or for the "price mix" or for interrelations of the price mix with the "product mix" and the "process mix" or the "materials mix". However, the Leontief system is useful in analyzing the output levels that would bring all the sectors into exact equilibrium for certain specified deliveries to final demand. Often the input-output model is criticised that it concentrates only on the "product mix" at the cost of all other mixes. With its simplified assumptions, the Leontief system lends itself to numerical computations using the data on the flows of goods between the different sectors of an economy. Let us now consider the similarities and differences between the Walrasian equilibrium and the input-output system.

Walrasian general equilibrium

Balderston (1954), while discussing the Walrasian equilibrium, has stated that Walras, in essense, showed how in a static equilibrium, under conditions of free competition, unknown prices and quantities of produced goods could be found, given the utility functions for individuals, coefficients of production and individual as well as market demand functions. Cassel (1932) however, assumed as given parameters the market supply and demand functions and Leontief's input-output framework is more like Cassel's rather than the Walrasian system.

Though Walras' contribution was, no doubt, great in its theoretical framework, he has been criticized for considering a static system. Further, he did not prove that the system of equations would have a unique solution. Goodwin (1953) has shown that the method of reaching equilibrium, once it is disturbed, may differ from the solution of the mathematical problem. It was Wald (1934) who set down the conditions for a unique solution. Wilfred Pareto (1907) introduced the concept of changes in the parameters in the general equilibrium. Further, refinements in the field of comparative statics have also been suggested by Hicks (1946) and Samuelson (1948).

The assumption of free competition is again an over-simplification of the existing system. In the terminology of Moore (1929), "The Walrasian equations are not deduced from reality but are hypothetical, and the equilibrium to which the mathematical conditions lead is an ideal statical equilibrium." The recent developments in the theory of games put forth by von Neumann and Morgenstern (1944) have opened new vistas for including the effects of monopoly, government intervention, et cetera in describing solutions reached by market.

Another criticism leveled against the Walrasian system is the assumption of linear homogenous production functions. This implies that the coefficients of production are fixed and no substitution of inputs is possible. Walras

(1954), however, has defended his system that under free competition, the profits are zero and that a simplifying assumption could be made that equal quantities are produced by each entrepreneur and that fixed and variable costs are proportional to the total costs. Variability of coefficients could be introduced within the framework of Walrasian system and Hicks (1946) and Samuelson (1948) have suggested minimizing the value of production factors (costs) subject to the production function in order to obtain the inputs used for various outputs. The input-output model also, in its simple form, assumes constant technical coefficients and free competition.

The input-output system

In the input-output model, the economy is divided into a number of industries which consume the products of other industries and their products again go as inputs of some other industries. In addition, in the open model, there is the exogenous final bill of goods, which uses up products of industry, but there is no output produced within the system. The open input-output model can be schematically shown as in Table 1.

To start with, in the Leontief input-output system we have a set of balance equations describing the input and output for all goods and services. The second relation is the set of constant technical input-output coefficients of the form $a_{ij} = x_{ij}/X_j$, where a_{ij} is the constant input coefficient,

Table 1. Scheme of input-output table

Consumers→ Producers↓	1	2	3	...	n	Final demand	Total output
1							
2							
3							
.							
.							
.							
n							

x_{ij} is the amount of goods flowing from sector i to sector j and X_j is the total output of the commodity of sector j. Thus, with a set of constant input-output coefficients and specified final demands (Y), the system of equations (given in detail in the following pages under the heading mathematical model) can be solved for the outputs required to satisfy the final demands.

Similarities and dissimilarities of the Walrasian and Leontief systems:

Walras has four sets of equations dealing with

(a) market supply and demand functions for factors

(b) market demand functions for goods

(c) services supplied equals demand

(d) cost of production equals prices of each good.

In the Leontief's model, the three sets of equations are:

(e) balance equations describing the output of each good and its uses as a consumption good, investment good and as inputs in further production

(f) the average cost equals price, and

(g) fixed technical relation between input and output.

Both Walras and Leontief assume static equilibrium as reflected in conditions (c) and (g). Similarly, they also assume free competition as could be seen from conditions (d) and (f). By assuming constant technical coefficients, Walras equates supply and demand of services and also cost and prices of individual goods (c and d) and likewise, Leontief also has fixed technical coefficients over short periods (g). In the input-output model, there are no market supply functions or demand functions for factors in terms of price as in the Walrasian system. There is also no counterpart for the individual utility functions of Walras in the input-output system and final demand is specified as a result of independent investigations or, in other words, it is exogenous to the model.

Carl Christ (1955) has stated that the input-output analysis can not be rightly designated as a general equilibrium system. While he does not dispute that the system is general, he hesitates to concede that it is an equilibrium system. He observes that input-output analysis is different from the general equilibrium theory in that it is not in itself an equilibrium system any more than is any other production function. The input-output system can be transformed into a general equilibrium system by introducing utility functions or demand schedules to reflect the preferences of the users of outputs. The introduction of optimizing on the demand side will yield a general equilibrium system. In the programming formulation developed in the later sections, the element of choice is introduced and the optimizing behavior is incorporated in the input-output system. Cornfield, Evans and Hoffenberg (1947) have built a general equilibrium system around the input-output production function. They first projected the labor force, wages and labor hours for 1950 and estimated full employment national income in 1950. Then, they estimated the quantities of goods and services that may be demanded by households and businesses from each of the industries at the expected level of income. Treating this as the final bill of goods, they projected total outputs. The choice factor enters in their model in their implicit use of the consumption and investment functions. We shall proceed to consider some of the aggregate growth models and their relation to the input-output framework.

AGGREGATE GROWTH MODELS AND THE INPUT-OUTPUT SYSTEM

One of the problems facing both the developed and less developed economies is the allocation of total investment funds, available from year to year, among the different sectors of the economy so as to ensure that the resultant growth of the economy is balanced in character. Several growth models have been developed to tackle this problem of investment and these models generally fall under one of two types, viz., aggregate growth models and disaggregate growth models. Aggregate growth models are generally based on the assumption of a single sector, comprising of the total national product of the economy, while in the disaggregate models, the economy is divided into several interdependent sectors for purposes of analysis.

In this section, we do not propose to consider the entire field of growth models, since it is outside the scope of our study, but we will only discuss the main aspects of the Harrod-Domar approach, which has close similarity to the Mahalanobis' model. As the Mahalanobis model provided the structural scaffolding for the Second Five Year Plan, we have considered it in greater detail. It will then be shown as to how the macro-models discussed are structurally similar to the dynamic input-output model. The final sections will deal with some of the planning models developed within the input-output framework.

The Harrod-Domar Model

The aggregate growth models essentially originate from the Harrod-Domar (1957) models, which in turn take their inspiration from Keynes (1935) insofar as the saving investment equilibrium and multiplier-accelerator mechanism is concerned. The models generally concentrate on investment rates, which will result in a steady economic growth of the country. Though these models have direct applicability in respect of mature, advanced economies, nonetheless they have been found useful for the formulation of economic policies for the less developed countries as well.

Assuming that a certain proportion of income is being saved, the mathematical setup of the Harrod-Domar model may be stated as

$$S_t = sY_t \tag{2.1}$$

where S_t is the saving at time t, s, the marginal propensity to save and Y, the national income at time t. It is assumed that the marginal propensity to save is equal to the average. Investment at time t results in an increment of national income at period t+1. If we denote the increment of income by ΔY_t and investment by I_t, the relationship between the two can be expressed as

$$\Delta Y_t = \beta I_t \tag{2.2}$$

where β is the reciprocal of the capital-output ratio, I_t, the investment at time t and ΔY_t, the increment in national income.

Under conditions of equilibrium, investment equals savings, which can be written as

$$I_t = S_t \tag{2.3}$$

It follows that

$$\frac{\Delta Y_t}{\beta} = sY_t \tag{2.4}$$

which can be expressed as

$$\frac{\Delta Y_t}{Y_t} = s\beta \tag{2.5}$$

Equation 2.5 denotes that the proportional rate of growth of income is equal to the saving coefficient multiplied by the output-capital ratio. Rewriting Equation 2.5 we have

$$\frac{Y_{t+1} - Y_t}{Y_t} = s\beta \tag{2.6}$$

or

$$Y_{t+1} = Y_t + s\beta Y_t = Y_t(1 + s\beta) \tag{2.7}$$

Solving the difference equation we get

$$Y_t = Y_0(1 + s\beta)^t \tag{2.8}$$

Equation 2.8 describes the equilibrium rate of growth of national income over time, given the marginal propensity to save (s) and the output-capital ratio (β).

Now if we assume the capital-output ratio to be constant, we can ascertain the amount of savings needed to realise a particular rate of growth. Denote the relative rate of growth as r. Then it follows that

$$s = r/\beta \tag{2.9}$$

or, in other words, the savings rate is equal to the rate of growth multiplied by the capital-output ratio. The growth of per capita income can be obtained from Equation 2.9 by subtracting the rate of increase of population from the rate of growth of total income, which can be expressed as

$$\bar{r} = s\beta - p \tag{2.10}$$

where $\bar{r}$ is the rate of growth of per capita income and p, the population growth. If β is assumed to be a constant, the saving ratio required for the desired increase of per capita income $\bar{r}$ may be derived as

$$s = \frac{1}{\beta}(\bar{r} + p)$$

or $$s = k(\bar{r} + p) \tag{2.11}$$

where k is the constant capital-output ratio.

The Harrod-Domar model, thus, indicates in an aggregative manner, the amount of savings required to raise the per capita income, given the rate of growth of population and the rate of growth of per capita income, and assuming constant capital-output ratio. The model, being highly aggregative, conceals many of the structural aspects of the problem of steady rate of growth.

The Mahalanobis' Model

The Mahalanobis' models are just an extension of the Harrod-Domar, which he developed independently. The models introduce additional parameters, which we will discuss later, and hence may be considered relatively more operational. Professor Mahalanobis (1953) for the sake of simplicity has developed a one sector model, where he assumed a population growth of 1-1/4 percent, a rate of net investment of about 5 percent and a national income coefficient of net investment of 30 to 33 percent (as high as in the United States of America), and showed that the rate of increase of per capita income in India will be about one-fourth of one percent, which is not significant for all practical purposes. If, however, the per capita income in India is to be doubled in, say, 35 years, with the population growth at the assumed rate, the per capita net national income must increase by 2 percent and the total net national income by three and a quarter percent. In order to attain this level, the rate of new investment should be of the order of 10 to 11 percent of net income per year.

In his two sector model, Mahalanobis (1953) extended the single sector of the economy to two sectors, the investment goods sector and the consumers' goods sector. He also assumes that investment (I_t) can be divided into two parts, one part going to investment goods sector (λ_k) and the other part to the consumption goods sector (λ_c). It follows that $\lambda_k + \lambda_c = 1$.

If β_k and β_c are the respective output-capital ratios of the two sectors, we can then express

$$I_t - I_{t-1} = \beta_k \lambda_k I_{t-1} \tag{2.12}$$

and

$$C_t - C_{t-1} = \beta_c \lambda_c I_{t-1} \tag{2.13}$$

Equation 2.12 can be solved as

$$I_t = I_0(1 + \lambda_k \beta_k)^t \quad , I_0 = \text{initial investment} \tag{2.14}$$

$$I_t - I_0 = I_0\{(1 + \lambda_k \beta_k)^t - 1\} \tag{2.15}$$

Similarly from Equation 2.13 it can be derived that

$$C_t - C_0 = \beta_c \lambda_c I_0 \frac{(1 + \lambda_k \beta_k)^t - 1}{\lambda_k \beta_k} \quad (2.16)$$

Adding up Equation 2.15 and Equation 2.16 we get

$$Y_t - Y_0 = I_0\{(1 + \beta_k \lambda_k)^t - 1\}\left(\frac{\beta_c \lambda_c}{\beta_k \lambda_k} + 1\right) \quad (2.17)$$

If we write $I_0 = \alpha_0 Y_0$, where α_0 is a certain constant less than unity denoting the proportion of investment to income, then we have

$$Y_t - Y_0 = \alpha_0 Y_0 \frac{\beta_c \lambda_c + \beta_k \lambda_k}{\beta_k \lambda_k} \{(1 + \beta_k \lambda_k)^t - 1\}$$

or

$$Y_t = Y_0 \left[1 + \alpha_0\left(\frac{\beta_c \lambda_c + \beta_k \lambda_k}{\beta_k \lambda_k}\right) \{(1 + \beta_k \lambda_k)^t - 1\}\right] \quad (2.18)$$

A comparison of Equation 2.18 with Equation 2.8 shows that the time path of income depends on a larger number of structural equations in the Mahalanobis' model than in the Harrod-Domar model. Several interesting conclusions emerge from the growth Equation 2.18. If we consider β's to be technologically fixed, then the growth of income would depend on α_0 and λ_k since λ_c is fixed once we know λ_k. Assuming α_0

to be a fixed constant, the planner is left with only one policy instrument viz., λ_k. A high value for λ_k renders the system to grow at slower rate in the initial stages and it gains momentum with passage of time. Chakravarty (1959) has also shown that for a high λ_k, the planned marginal rate of savings must also be higher. Haldane (1955) has, however, shown that the optimum value of λ_k may be derived by maximizing national income or its growth over the specified planning period.

Apart from the problem of optimum allocation of investment funds, there was acute unemployment in the country and hence the planners had to necessarily consider employment as one of the important targets, while formulating the Second Five Year Plan. Mahalanobis (1955) extended his two sector model to one of four sectors to meet the dual objective of maximum national income, compatible with specified employment opportunities. The four sectors in his model are (1) investment goods industries (k), (2) factory organized consumers' goods industries (k_1), (3) small scale household industries producing consumers' goods (k_2), and (4) service industries including health, education, et cetera (k_3). If we denote N_k, N_1, N_2 and N_3 as the number of additional persons employed in the four sectors and if we describe A as the total investment, N the total number of persons employed and E the total increase in income over the period, the following relationships are established:

$$N = N_k + N_1 + N_2 + N_3 \tag{2.19}$$

$$A = N_k\theta_k + N_1\theta_1 + N_2\theta_2 + N_3\theta_3 \tag{2.20}$$

$$E = \beta_k N_k\theta_k + \beta_1 N_1\theta_1 + \beta_2 N_2\theta_2 + \beta_3 N_3\theta_3 \tag{2.21}$$

$$Y_t = Y_0\{(1 + \eta)^t - 1\} \tag{2.22}$$

where θ is the net investment per employed person and other parameters are the same as defined earlier. If a constant annual rate of growth of income, say at η percent per annum is assumed, then E could be derived from initial income Y_0.

Sectoral coefficients and growth indicators

At this stage, we might consider some of the sectoral coefficients of the Mahalanobis' model. Several published sources, as the estimates of the agricultural production by the Central Statistical Organization and the Index of Industrial Production are available to provide a basis for the estimation of national income. Uma Datta (1961) has estimated the levels of net domestic product in 1955-56 and 1960-61 at the 1958-59 prices as is shown in Table 2.

From Table 2 the rise in national income is of the order of 18 percent based on the estimates of the national income taking into account the actual levels of production.

Table 2. Net domestic product at 1958-59 prices (in 100 crores of rupees)

Sector	1955-56	1960-61	Percent increase during 1956-61
1. Agriculture, animal husbandry, etc.	55.7	62.1	11.4
2. Mining, manufacturing and small enterprises	20.1	25.5	26.8
3. Commerce, transport and communications	18.8	22.4	19.1
4. Professions and services including government administration	17.3	22.3	28.9
5. Net domestic product and factor cost	111.9	132.3	18.2

Using the figures of income, investment and employment, she has calculated the sectoral values of the ratio of increment of income to investment (β) and the net investment required per person (θ) for the Second Five Year Plan period and has compared them with the figures assumed by Mahalanobis (1955), as shown in Table 3.

Table 3 is a broad indicator of the country's economic conditions. However, one notable feature is that the overall average incremental capital-output ratio works out to 3.3:1, as against 2.3:1 estimated in the Second Five Year Plan. To what extent this high ratio is due to the capital intensive

Table 3. Sectoral coefficients

Sector	Incremental output-capital ratio (β)		Net fixed investment per engaged person (θ)	
	At the end of 2nd Plan	Assumed by Mahalanobis	At the end of 2nd Plan	Assumed by Mahalanobis
1. Basic investment goods	0.15	0.20	25,000	20,000
2. Large scale consumer goods	0.25	0.35	14,200	8,750
3. Agriculture and small scale industries	0.60	1.25	5,515	2,500
4. Services	0.30	0.45	6,550	3,750

Table 4. Sectoral coefficients in the Third Plan

Sector	β's	θ's
1. Basic investment goods (k)	0.20	25,000
2. Large scale consumer goods (c_1)	0.35	10,000
3. Agriculture and small scale industries (c_2)	0.75	5,500
4. Professions and services	0.45	6,000

nature of the sectors or otherwise cannot be accurately assessed, because the price changes between 1952-53 and 1958-59 have been considerable and no details are available separately on the price changes of products and investment.

<u>Programming formulations of Mahalanobis model</u>

Uma Datta (1961) has also formulated the linear programming framework of the Mahalanobis' model to determine the possible rise in national income during the Third Plan period, i.e. 1960-61 to 1965-66. The programming formulation can be represented as

$$\text{Max} \quad \theta_k \beta_k N_k + \theta_{c_1} \beta_{c_1} N_{c_1} + \theta_{c_2} \beta_{c_2} N_{c_2} + \theta_{c_3} \beta_{c_3} N_{c_3} \tag{2.23}$$

subject to

$$N_k + N_{c_1} + N_{c_2} + N_{c_3} = N \tag{2.24}$$

$$\theta_k N_k + \theta_{c_1} N_{c_1} + \theta_{c_2} N_{c_2} + \theta_{c_3} N_{c_3} = I \tag{2.25}$$

$$\text{and} \quad \theta_k N_k = \lambda_k I \tag{2.26}$$

where N is the total additional employment to be generated during the Third Plan, I, the net investment and λ_k, the proportion of total investments in the producers' goods industries. k is the basic investment goods industry, c_1 is the large scale consumer goods industry, c_2 is the agriculture and small scale industries and c_3 is the services sector. Subscripts denote the particular sectors and absence of subscripts indicates national parameters.

The values of parameters used in the above programming formulation are given in Table 4 on page 27. The simplex solution using the parameters in Table 4 indicated that with a fixed investment of 9600 crores, an increase in national income of 44 percent can be achieved, generating additional employment of 14 million workers. But the solution also indicated that 50 percent of total investment should be in agricultural and small enterprises, with no investment in services sector. The problem was reformulated in order to attain a balanced growth and the second best maximum was obtained by splitting the services sector as the capital intensive portion and less capital intensive part. An additional constraint was also imposed that

$$N_k + N_{c_1} \leq N_{c_2} \tag{2.27}$$

which implies that employment generated in the factory sector is less than the employment in agriculture and small scale industries. The simplex solution with the additional constraints is given in Table 5.

The solution in Table 5 gives a 36 percent rise in national income and an investment of 1727 crores in agricul-

Table 5. Simplex solution

Sectors	Additional employment (in millions)	Fixed investment			
		As estimated		As in draft outline	
		Rupees crores	Percent of total	Rupees crores	Percent of total
1. Investment goods (k)	0.84	2100	22	2100	22
2. Factory consumer goods (c_1)	2.30	2300	24	1525	16
3. Agriculture and small scale industries (c_2)	3.14	1727	18	2565	27
4. Services (c_3)	7.72	3474	36	3410	35
Total	14.00	9601	100	9600	100

ture and small scale industries and 3474 crores in the services sector. The growth of the economy over the Third Plan period is found to be of the order of 7 percent per annum and the pattern of investment is broadly in line with the draft plan. We have discussed the linear programming framework of Mahalanobis' model developed by Uma Datta (1961) at some length in order to show the differences between the parameters in this program and in the empirical model to be developed by the author later within the input-output framework.

Having considered the Mahalanobis (1955) planning model and the linear programming framework of Uma Datta (1961) using the same parameters as Mahalanobis proposed, let us now consider some of the weaknesses of the model. Komiya (1959) has criticized that the Mahalanobis' model seems to neglect the demand side of economic planning. Secondly, he shows that the increase in national income can be more than Mahalanobis' solution and therefore, it is not an optimum allocation of resources as claimed by him. Thirdly, the model pays no heed to the problem of factor prices and when possible patterns of factor prices are taken into account, the accuracy of the parameters seems to be in doubt.

Komiya (1959) has analysed the Mahalanobis model using the same coefficients used by Mahalanobis. The initial national income is taken at 108,000 million rupees per annum, the total investment funds at 56,000 million rupees during the Second Five Year Plan period and the target for new employment at 11 million jobs. The increase in national income per year at the assumed rate of 5 percent is nearly 29,000 million $\{I + 0.05)^5 - I\}$. The investment in sector 1, i.e., investment goods sector, can be calculated as 1/3 of 56,000 million or 18,500 millions of rupees. Increase in income produced in that sector can be obtained by dividing the investment by the capital-output ratio, i.e., $18{,}500 \div 5 = 3700$ million. Increase in employment in sector 1 is obtained by multiplying the labor in man years by the increase in income or roughly $3700 \times .00025 = 0.9$ million. Now the planning problem of Mahalanobis is reduced to the distributing the remaining investment fund of 37,500 million rupees among the rest of the sectors so as to increase income by 25,300 million rupees and provide employment to the balance of 10.1 million people. The deterministic solution of Mahalanobis may be obtained from the following set of equations. The system of equations may be written as

$$\Delta Y = Y_2 + Y_3 + Y_4 = 25300 \tag{2.28}$$

$$\Delta K = a_2Y_2 + a_3Y_3 + a_4Y_4 = 37500 \tag{2.29}$$

$$\Delta N = b_2Y_2 + b_3Y_3 + b_4Y_4 = 10.1 \tag{2.30}$$

where Y_i is the increase in national income of the i th sector, ΔY, ΔK and ΔN the total increase in national income, capital stock and employment. a_i is equal to $1/\beta_i$ (β_i defined as the increase in net national income per unit of time to the net investment necessary for it) and b_i is equal to $1/\theta_i\beta_i$ (θ_i being the ratio of the net investment to corresponding increase in employment).

The above model has been criticized by Komiya (1959) that it completely neglects the demand side. He points out that the total demand for consumers' goods will be determined by the marginal propensity to consume, but in the model, the total supply of consumers' goods and the supply of each group of consumers' goods is determined only from the considerations of supply and the level of demand has not been taken into account. Again, supply and demand has not been equated and the balance of payments problem has not been taken care of in the model. The model also pays no specific attention to the supply and demand of intermediate products, as also replacement of capital goods. The model can be useful for economic planning only if all the restrictive conditions are taken into account.

Instead of assuming 5 percent increase in national income, Komiya maximizes the national income under the given conditions of the supplies of investment funds and labor force. The programming problem may be written as

$$\text{Max } \Delta Y = Y_2 + Y_3 + Y_4 \tag{2.31}$$

subject to

$$a_2Y_2 + a_3Y_3 + a_4Y_4 \leq 37,500 \tag{2.32}$$

$$b_2Y_2 + b_3Y_3 + b_4Y_4 \leq 10,100,000 \tag{2.33}$$

$$Y_2 \geq 0, \quad Y_3 \geq 0, \quad Y_4 \geq 0$$

where Y_i, a_i and b_i are the same as defined in Equations 2.28 through 2.30 above. The solution suggested that all the capital should be invested in sector 3 and about a third of the capital is not used up, while all the labor is used up. Since this is contrary to theoretical expectations, he changed the weights in the maximizing function allowing for subsidies and taxes in factor requirements. The reformulated problem yielded a solution in which national income was higher than that estimated by Mahalanobis from his model. Hence, Komiya concluded that the solution emanating from the Mahalanobis model is not "an optimum allocation of resource" as claimed by him.

Thus, Komiya (1959) showed that instead of developing a deterministic model, a model with some choice always leads to an optimum solution, given the set of restrictions.

Macro Economic Models and the Dynamic Input-output System

Having considered some of the macro models used for planning, it might be useful to investigate the feasibility of casting these models in the Leontief sectoral framework. Mathematical models may be divided into two types, viz. static and dynamic models. Dynamic models are characterized by functional equations, which involve variables relating to different points of time (dated variables). Static models are the limiting cases of dynamic models, where the variables relate to a particular point of time (undated variables). The general equilibrium of the Lausaune school, followed by later economists like Hicks, Samuelson, Lange and others, is a classic example of a static mathematical model.

The static Leontief system is concerned with problems of static compatabilities of levels of production in different sectors of the economy, if the specified bill of goods is to be produced. The mathematical model adopted by Leontief is just a simplified version of the general equilibrium system. Economists are generally concerned with models of economic processes or activities, which throw some light in planning long term economic development and here the dynamic version is more useful than the static one. Therefore, we may consider the analogy of the Leontief dynamic model with that of the Harrod-Domar model. Leontief's dynamic system differs from the static system by the introduction of stock flow coefficients.

Let S_{ij} be the output of the i th industry held by j th industry as stock and X_j be the gross output of the j th industry, and b_{ij} indicate the size of stock of the commodities required per unit of commodity j or the capital coefficient of commodity i in industry j. Assuming a relationship

$$S_{ij} = b_{ij} X_j \tag{2.34}$$

it follows that

$$\dot{S}_{ij} = b_{ij} \dot{X}_j \tag{2.35}$$

where $\dot{S}_{ij}$ and $\dot{X}_j$ denote the rate of change of the variables with respect to time.

Now the balance equations of the Leontief system may be written as

$$X_i - \sum_{j=1}^{n} X_{ij} - \sum_{j=1}^{n} \dot{S}_{ij} = F_i \tag{2.36}$$

$$(i = 1, 2, \ldots, n)$$

where F_i is the final demand and other variables are as defined earlier.

By substituting Equation 2.35 into 2.36 we get the balance equation as

$$X_i - \sum_{j=1}^{n} a_{ij} X_j - \Sigma \; b_{ij} \dot{X}_j = F_i \qquad (2.37)$$

$$(i = 1, 2, \ldots, n)$$

If $F_i = 0$, then we get the closed version of the dynamic model. The dynamic Leontief model could be shown to be analogous to the Harrod-Domar model.

The basic equation of Harrod-Domar may be written as

$$sY_t = \frac{\Delta Y_t}{\beta} \qquad (2.38)$$

where s is the saving coefficient and is equal to 1 - a and a is the consumption coefficient. β is the reciprocal of the capital-output ratio and let $1/\beta$ be equal to b.

Equation 2.38 may be rewritten as

$$(I - a)\, Y_t = b\, (Y_{t+1} - Y_t) \qquad (2.39)$$

The solution of this difference equation will give an income growth equation over time.

Considering the balance Equation 2.37 of the Leontief system and using finite differences, the equation can be written as

$$X_i = \sum_{j=1}^{n} a_{ij} X_j + \sum_{j=1}^{n} b_{ij} \Delta X_j \tag{2.40}$$

(since $F_i = 0$)

$$X_i - \sum_{j=1}^{n} a_{ij} X_j = b_{ij} \Delta X_j \tag{2.41}$$

In the matrix notation this could be compactly expressed as

$$(I - a) \{X\} = [b] \{\Delta X_t\} = [b] \{X_{t+1} - X \} \tag{2.42}$$

where I is the unit matrix, [a] the matrix of input-output coefficients, [b] the matrix of intersectoral capital-output ratios. It could be seen that the dynamic model expressed in Equations 2.39 has close similarity to Equation 2.42 except for the fact that scalars have been replaced by vectors. However, Leontief assumes matrix [a] to be technologically fixed, while in the Harrod-Domar model, the saving ratio can be altered to attain a certain specified rate of growth.

A Short Term Planning Model for India

Padma Desai (1961) has presented a planning model for the Indian economy by formulating an input-output model closed with respect to all household consumption except that originating from government employees. The distribution of consumption expenditure among the households is determined endogenously, each group having a specific consumption pattern.

The endogenous sectors are (i) agricultural, (ii) manufacturing, (iii) services and (iv) transport and (v) household sectors. There are as many household sectors as processing sectors. Each household sector derives its income from the corresponding processing sector. The exogenous sectors create demand for exports, capital formation, government outlays and consumption expenditure of government employees. The household now includes wages and salaries and distributed dividends. The matrix of intersectoral transactions is presented in Table 6.

The consumption expenditure of households in each sector is derived by subtracting taxes and savings from the household row to ascertain the disposable income. The disposable income is distributed according to the consumption expenditure of each household sector on different commodities and services. For this purpose, a pattern of consumer expenditure of rural, urban and mixed households is drawn up on the basis of data available in the All India National Sample Surveys. The items of expenditure of the households in the different sectors are given in an abridged form in Table 7.

The rural pattern of consumption expenditure in Table

Table 6. Intersectoral transfers of intermediate products and commodities and services for final consumption 1950-51 (in rupees billion)

Outlays in → for purchases from ↓	Agriculture	Manufacturing	Services	Trade and transport	Foreign countries (exports to)	Government	Gross capital formation: Private	Gross capital formation: Government
Agriculture	11.17	12.247	----	.98	.613	-----	-----	
Manufacturing	1.86	8.329	.433	1.39	2.160	3.08	2.650	1.362
Services	-----	.117	----	.49	----	1.090	-----	-----
Trade and transport	.632	3.969	.091	.306	.139	.625	.530	.272
Foreign countries (imports from)	.158	2.834	.021	.264	----	.053	-----	-----
Government	-----	-----	----	----				
Depreciation allowances and retained surplus	-----	1.161	.183	.23		.19		
Households	47.34	14.199	6.448	14.75		5.36	-----	.816

Table 6 (Continued)

Outlays in → for purchases from ↓	Households in					Total gross output	Exogenous demand
	Agriculture	Manufacturing	Services	Trade and transport	Government		
Agriculture	25.022	6.119	2.318	4.851	1.765	64.905	2.378
Manufacturing	9.723	3.072	1.587	3.641	1.363	40.650	10.615
Services	3.50	1.27	.78	1.78	.67	9.697	1.760
Trade and transport	4.916	1.950	.944	2.145	.794	17.317	2.360
Foreign countries (imports from)	.799	.229	.101	.233	.088		
Government	-----	-----	-----	-----	-----	------	
Households						88.913	

Table 7. Pattern of consumer expenditure

Items of expenditure	Percent of total expenditure by households		
	Rural	Urban	Mixed
Agriculture	61.9	43.8	55.8
Manufacturing	28.8	37.7	31.9
Services	7.8	16.0	10.6
Trade and transport	1.3	2.6	1.7
Total	99.8	100.1	100.0

7 is applied to agricultural households and the urban pattern to households in services, trade and transport and government sectors. The manufacturing sector is divided into large scale industries, which have the mixed pattern and small scale industries, which is likely to have the rural pattern. Allowing for taxes and trade margins, the expenditure of each household is computed in terms of producers' prices.

Generalized model

Consider m processing sectors and m household sectors, each household deriving its income from the corresponding processing sector. Let the processing sectors be identified from 1 to m and the household sectors (m + 1) to (m + m). The gross output is represented by q_1, q_2, q_3, ..., q_m, q_{m+1}, q_{m+2}, ..., q_{m+m}. The output of each sector is distributed to the m processing sectors, household sectors and to exogenous demand.

The input-output coefficients are of the form

$$q_{ij} = a_{ij}\, q_j \tag{2.43}$$

$$(i = 1, 2, \ldots, m, m+1, m+2, \ldots, m+m)$$
$$(j = 1, 2, 3, \ldots, m)$$

It is also assumed that for each household sector, the consumption expenditure is distributed in a fixed proportion of its earnings.

$$h_{i,m+j} = a_{i,m+j}\, q_{m+j} \tag{2.44}$$

$$(i = 1, 2, 3, \ldots, m)$$
$$(j = 1, 2, 3, \ldots, m)$$

where $h_{i,m+j}$ is the expenditure of households in the m + jth sector on consumer goods and services of i th sector; $a_{i,m+j}$, the consumption coefficient, shows the amount of consumers goods and services of the i th sector purchased with a unit of household services in the m+j th sector and q_{m+j} is the earnings of the households in the m+j th sector.

$$q_i = q_{i1} + q_{i2} + \ldots + q_{im} + a_i \tag{2.45}$$

$$(i = 1, 2, 3, \ldots, m, m+1, m+2, \ldots, m+m)$$

where q_{ij} is the flow from sector i to the j th processing sector and a_i is its flow to household sectors and final demand. Substituting yields,

$$q_i = \sum_{i}^{m} a_{ij} q_j + \sum_{m+1}^{m+m} a_{ij} q + X_i + G_i + I_i \qquad (2.46)$$

where X_i is imports, G_i government expenditure, I_i the amount of sector i output that moves into gross capital formation. The matrix of all intersectoral flows is represented as shown on page 45. The matrix of input-output coefficients and consumption coefficients from Table 6 may be represented as shown on page 46. The upper left submatrix on page 45 contains input coefficients a_{ij} defined in Equation 2.43. The lower left-hand submatrix contains $a_{m+1,j}$ coefficients with regard to household sectors. As the household sector receives its income from the corresponding processing sector only, non-diagonal elements are zero. The upper right-hand coefficients contain $a_{i,m+j}$ defined in Equation 2.44. The lower right-hand matrix is all zero since there are no intersectoral flows.

The processed outputs q_1, q_2, ..., q_m and the household incomes q_{m+1}, q_{m+2}, ..., q_{m+m} can be expressed as sums of purchases by all endogenous sectors in the form

$$Q = A \cdot Q + Y \qquad (2.47)$$

$$
\begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1m} & a_{1,m+1} & a_{1,m+2} & \cdots & a_{1,m+m} \\
a_{21} & a_{22} & \cdots & a_{2m} & a_{2,m+1} & a_{2,m+2} & \cdots & a_{2,m+m} \\
 & \vdots & & & & \vdots & & \\
a_{m1} & a_{m2} & \cdots & a_{mm} & a_{m,m+1} & a_{m,m+2} & \cdots & a_{m,m+m} \\
 & & & & & & & \\
a_{m+1,1} & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\
0 & a_{m+2,2} & \cdots & 0 & & \vdots & & \\
 & \vdots & & & & \vdots & & \\
0 & 0 & \cdots & a_{m+m,m} & 0 & 0 & \cdots & 0
\end{bmatrix}
\qquad (2.48)
$$

$$
\begin{bmatrix}
0.1721 & 0.3013 & 0.0000 & 0.0566 & 0.5286 & 0.4309 & 0.3316 & 0.3289 \\
0.0287 & 0.2049 & 0.0447 & 0.0803 & 0.2054 & 0.2165 & 0.2461 & 0.2468 \\
0.0000 & 0.0029 & 0.0000 & 0.0283 & 0.0739 & 0.0894 & 0.1210 & 0.1207 \\
0.0097 & 0.0975 & 0.0094 & 0.0177 & 0.1038 & 0.1373 & 0.1464 & 0.1454 \\
 & & & & & & & \\
0.7924 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0.3493 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.6649 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.8522 & 0 & 0 & 0 & 0
\end{bmatrix}
\tag{2.49}
$$

where Q is the output vector of order m+m x I, Y is the exogenous demand vector of order m+m x I, the first m elements of Y are $X + G + I$, where $i = 1, 2, 3, \ldots, m$ and the remaining m elements are zero.

The solution $Q = (I - A)^{-1} Y$ gives the values of the m processed outputs and m household income as a function of exogenous variables Y. The inverted matrix gives the output and income requirements for any number of alternate exogenous vectors. For example, one can find out the distribution of processing and household sectors of the 'additional' output generated through a billion rupee increase in gross capital formation. Likewise, if the plans call for a billion rupee cut in agricultural exports and that resources can be transferred to meet an additional final demand for services of 1.4 billion on government account, the increase of household income in services could be directly read from upper right-hand submatrix of System 2.49. Thus, within the framework of input-output technique, Padma Desai has provided a short term planning model for India. Let us now consider a long term planning model using the input-output coefficients.

A Long Term Planning Model

The long term objective of the Planning Commission is to divide the process of development into shorter plans, like the Five Year Plans, so that certain priorities could be set up in reaching the ultimate goals. However, the process of planning is to be considered as a whole and the objectives of each shorter planning period are linked with a larger perspective. Thus, the First, Second and Third Plans should be conceived of as stages in the long term economic development of the country.

The macro models, as the Harrod-Domar type, are highly aggregate models and for a planner, information on a number of small disaggregate sectors might be needed to assess the different magnitudes of sectoral development in the process of planning. With this end in view, Sandee (1960) built a long term planning model for India, distinguishing thirteen sectors. In the model, he utilized the input-output coefficients of the inter-industry table for 1953-54 with certain modifications, wherever necessary.

Taking into account the general principles of aggregation, discussed elsewhere, Sandee (1960) adopted mostly the input-output coefficients from the 1951-52 and 1953-54 inter-industry tables for India. Wherever the coefficients are known to change considerably, he modified those coefficients to account for the possible changes. For example, there was reason to assume that electricity consumption per unit of output was expected to go up by one hundred and fifty-two percent during the Second Five Year Plan. He foresaw the trend to continue in the Third and Fourth Plans.

In order to account for these changes, he multiplied the input-output coefficients for electricity by four, or in other words, the electricity input coefficients were taken four times higher than given in the table for 1953-54. Similar adjustments were made for the steel sector also. In the model, he identified the following sectors:

21. Agriculture (including plantations, fishing and small scale food industries
22. Large scale food manufacturing
23. Steel industry
24. Electrical power industry (both thermal and hydro)
25. Coal mining
26. Fertilizer industry (nitrogenous fertilizer only)
27. Transport
28. Heavy engineering
29. Other equipment industry
30. Other large scale industries (including other mining)
31. Construction (including the cement and small scale building materials industries)
32. Small scale industries
33. Housing

There is no separate sector as fertilizer industry (Sector 26) in the inter-industry table for India. It was built up from the project reports for new fertilizer plants. The first twelve balance equations are given in Table 8. No balance equation was set up for x_{33}, since we can treat

Table 8. Balance equations

$x_{21} =$	$0.44x_{22}$							
$x_{22} =$	$0.06x_{22}$							
$x_{23} =$						$0.01x_{27}$	$+ 0.30x_{28}$	+ 0
$x_{24} =$	$0.01x_{22}$	$+ 0.08x_{23}$	$+ 0.05x_{24}$	$+ 0.07x_{25}$	$+ 0.15x_{26}$	$+ 0.02x_{27}$	$+ 0.02x_{28}$	+ 0
$x_{25} =$		$0.10x_{23}$	$+ 0.10x_{24}$	$+ 0.05x_{25}$	$+ 0.07x_{26}$	$+ 0.03x_{27}$	$+ 0.05x_{28}$	+ 0
$x_{26} =$								
$x_{27} =$		$0.22x_{23}$			$+ 0.04x_{26}$	$+ 0.01x_{27}$		+ 0.
$x_{28} =$								
$x_{29} =$								+ 0.
$x_{30} =$	$0.04x_{22}$	$+ 0.20x_{23}$	$+ 0.11x_{24}$	$+ 0.07x_{25}$	$+ 0.18x_{26}$	$+ 0.16x_{27}$	$+ 0.13x_{28}$	+ 0.
$x_{31} =$								
$x_{32} =$								

	$+ 0.19x_{30}$	$+ 0.01x_{31}$	$+ 0.05x_{32}$	$+ c_{21}$	$+ e_{21}$	$+ n_{21}$		(2a·1)
	$+ 0.02x_{30}$			$+ c_{22}$	$+ e_{22}$	$+ n_{22}$		(2a·2)
$+ 0.18x_{29}$	$+ 0.04x_{30}$	$+ 0.07x_{31}$	$+ 0.01x_{32}$		$+ e_{23}$	$+ n_{23}$		(2a·3)
$+ 0.02x_{29}$	$+ 0.04x_{30}$	$+ 0.01x_{31}$					$+ 300$	(2a·4)
$+ 0.01x_{29}$	$+ 0.01x_{30}$	$+ 0.01x_{31}$				$+ n_{25}$		(2a·5)
					e_{26}	$+ n_{26}$	$+ x_{26-21}$	(2a·6)
$+ 0.01x_{29}$	$+ 0.01x_{30}$	$+ 0.09x_{31}$		$+ c_{27}$				(2a·7)
				$+ i_{28}$	$+ e_{28}$	$+ n_{28}$		(2a·8)
$+ 0.03x_{29}$				$+ i_{29}$	$+ e_{29}$	$+ n_{29}$		(2a·9)
$+ 0.10x_{29}$	$+ 0.15x_{30}$	$+ 0.11x_{31}$	$+ 0.22x_{32}$	$+ c_{30}$	$+ e_{30}$	$+ n_{30}$		(2a·10)
		$0.13x_{31}$		$+ i_{31}$		$+ n_{31}$		(2a·11)
		$0.06x_{31}$	$+ 0.04x_{32}$	$+ c_{32}$		$+ n_{32}$		(2a·12)

$x_{33} = c_{33}$ and thus save one equation.

The first twelve equations described in Table 8 are of the Leontief type. Equation 2a.13 explains the rise in agricultural output consequent on the increase in fertilizer applied ($x_{26.21}$), irrigation projects executed ($i_{31.21}$) and agricultural extension (i_{34}). Thus the equation for agriculture was derived from extraneous information on the possible course of development. The need for such a special treatment for the agricultural sector according to Sandee was because there was no fixed relation between the inputs and agricultural output. Another feature of the equation for agriculture is the inclusion of the effects of agricultural extension. Sandee included the extension work as the 'stimulant' for agricultural production of about 3.3 million tons annually, which is not accounted for in the increases as a result of fertilizer or irrigation. In functional form, the thirteenth equation for agriculture may be written as

$$x_{21} = f(x_{26.21}, i_{31.21}, i_{34}) \qquad (2a.13)$$

In addition to the thirteen equations, the following three equations are specified in the model to describe investment, exports and consumption.

$$I = i_{28} + i_{29} + i_{31} + i_{34} + \sum_{j=21}^{32} n_j \qquad (2a.14)$$

$$e_{21} + e_{22} + e_{23} + e_{26} + e_{28} + e_{29} + e_{30} = 0 \quad (2a.15)$$

$$C = c_{21} + c_{22} + c_{27} + c_{32} + c_{33} \quad (2a.16)$$

where I denotes total investment and n_j refers to rise in stocks in the sectors. Equation 2a.15 assures that the balance of visible trade between 1960 and 1970 will be more or less similar. Equation 2a.16 defines the total consumption of goods, transport and housing services C.

Constraints

Several constraints were imposed to regulate the export surplus. The constraints on export surpluses are

1. Agricultural export surplus	e_{21}	≤ 94
2. Food export surplus	e_{22}	≤ 177
3. Steel export surplus	$-e_{23}$	$\leq 0.25\ x_3 + 30$
4. Fertilizer export surplus	$-e_{26}$	≤ 350
5. Heavy engineering export surplus	$-e_{28}$	$\geq 0.25\ x_{28}$
6. Other equipment	e_{29}	≤ 92
7. Other large scale manufacturing	e_{30}	≤ 0

For illustration, we will indicate as to how the constraint on agricultural export surplus was established.

An upper limit to net exports was set by the elimination of imports of food grains and a considerable improvement in the balance of cotton exports and imports. An improvement by rupees 94 crores seemed the maximum that could be expected. Similarly, the upper limit to manufactured food was established by estimating an increase of 20 percent or 27 crores in tea exports, an improvement in the sugar balance by 50 crores and increase in oil exports to the tune of 100 crores, totaling 177 crores. In the same manner, constraints on other sectors were built up based on possible trends and other guesses on the course of development. The lower constraints on exports were constructed on the estimated world total supply and the quantities that could be purchased by India without upsetting the world market.

Equation 2a.16 defined welfare as the sum of six types of consumption viz. agriculture, large scale food manufacturing, transport, large scale industries, construction of residential property and small scale industries. As in the case of exports, lower and upper limits were built up for consumption also. The lower and upper constraints were set at 13 percent of 1960 consumption above and below Engel curves for each commodity.

Besides constraints on exports and consumption, constraints were also built up for investment and agricultural stimulants. The constraint on investment was worked out based on stock-flow coefficients. By the conventional procedure, total investment has been linked to the total income and the marginal propensity to save was estimated at 24.5 percent. Since the total material consumption c is taken as given in the model, it was estimated that for every crore of rupees increase in consumption, investment rises by $0.245/1 - 0.245 = 0.32$. The constraint for investment is, therefore, stated as

$$I = .32c \quad , \quad I \geq 0$$

Output-investment constraints (n) were set up for a number of individual sectors, as agriculture, large scale food, steel, fertilizer, transport, heavy engineering, other engineering and other large scale manufacturing. Investment in 1960 was linked with the growth rate of output in that year and the assumption of a linear trend in all investment flows over time was introduced. For example, the rate of growth of food manufacturing was put at rupees 25 crores per annum and the total increase in output from 1960 to 1970 should be at least 5 times this amount, viz. 125 crores of rupees. Instead of a constraint on the output of agriculture, a lower limit on irrigation was prescribed, which cannot fall below zero.

The model enables us to maximize a linear function of the variables in the 16 equations. With the help of constraints the welfare function, *viz.*, consumption is maximized by the linear programming technique. The levels of production, consumption and exports are calculated from the solution.

In the model, the level of material consumption in 1970 is used as the only criterion. As investment and welfare go together, maximizing consumption also indirectly amounts to maximizing investment. Further, it has been assumed in the model that savings would continue to restrict investment even after 1970 and so the target function was not modified. Sandee has also stated that if employment had also been taken into account, the plan would have been different. It is possible that some material welfare would be sacrificed to ensure more employment. Thus, using input-output coefficients for 1953-54 for the most part and building additional constraints on exports, consumption and investment, Sandee has projected the optimum output, consumption, investment and exports for 1970.

Economic development and the Leontief open model

In the open models, generally the household sector is treated as exogenous. While discussing the paper of Fox and Sengupta (1961), Richard Day* has suggested in a private communication the inclusion of household sector as endogenous

*Day, Richard, United States Department of Agriculture, Washington, D.C. Comments on Fox and Sengupta's paper on uses of the input-output and related techniques in partial analysis. Private communication. 1962.

to the model, similar to the approach of Padma Desai (1961), discussed earlier. By using the aggregated tables provided by Fox and Sengupta (1961), Day has computed that in the United States, a 10 percent increase in deliveries to final demand in 1947 of the industrial sector would call for an increase of 4-1/2 percent in agricultural output. This contrasts with a corresponding increase of 0.6 percent when the households are exogenous. Similarly, an increase of 10 percent to final demand of the agriculture complex would call for an increase of nearly 6 percent in the output of all other industries, as against 0.9 percent when households are exogenous. By this illustration, he demonstrates how the specification error can arise when important sectors are omitted from the Leontief model.

It is also pointed out by Day that the inclusion of the household sector as a part of the structure of the input-output model generates substantial demand for agricultural products to meet increased final deliveries to the industrial sector for investment purposes. This suggests that economic development of agriculture may be an important correlative for any industrial development program. Having considered different macro models, we proceed to study the mathematical structure and simple numerical applications of the input-output model.

THE LEONTIEF OR INPUT-OUTPUT SYSTEM

In describing the input-output analysis, Wassily Leontief (1951) admits that the input-output approach to the empirical analysis of inter-industrial relationships represents an admittedly crude attempt to combine facts and theory in the study of the structure and operation of a modern economy. Any economy can be described as a system of mutually inter-related industries or interdependent economic activities. The inter-relation consists in the steady stream of goods which links directly or indirectly all the sectors of the economy to each other. The inputs of one industry are the outputs of another and vice versa. These flows can be observed and described in quantitative terms.

The Leontief system may be an open system of linear equations describing the flows of commodities between the different sectors of an economy or a closed system. In the open system, household consumption of goods and services, capital formation, government, foreign trade and stocks form the components of final demand or the autonomous sectors. If all the sectors are both producers and consumers, the system is described as a closed one. Let us now consider the quantitative illustration of an open static Leontief system.

Mathematical Model

The open Leontief system describes the flow of commodities between the different sectors of an economy. These sectors are divided into two types. One is known as the intermediate sectors and includes those whose demand for the commodities of other sectors arises directly from their own decisions to produce goods. The other kind, namely, the final demand sectors are treated as autonomous sectors. The flow of goods to the intermediate and final demand sectors may be described in the form of equations as

$$
\begin{array}{ccccccccccccccc}
x_{11} & + & x_{12} & + & \cdots & + & x_{1j} & + & \cdots & + & x_{1n} & + & Y_1 & = & X_1 \\
x_{21} & + & x_{22} & + & \cdots & + & x_{2j} & + & \cdots & + & x_{2n} & + & Y_2 & = & X_2 \\
\vdots & & \vdots & & & & \vdots & & & & \vdots & & \vdots & & \vdots \\
x_{n1} & + & x_{n2} & + & \cdots & + & x_{nj} & + & \cdots & + & x_{nn} & + & Y_n & = & X_n
\end{array}
\qquad (3.1)
$$

where x_{ij} is the amount of output of sector i purchased by sector j, Y_i is the final demand for product i, and X_i is the total output of sector i and Y_i final demand for the i th producing sector.

The intersectoral flows, x_{ij}'s, may be in physical quantities or in value terms. Again, some transaction matrices are prepared taking into account the producers' prices, while others adopt market prices and thus, the valuational aspect varies with the conceptual framework and the nature of availability of data in the different countries. The system of equations in 3.1 can be represented in matrix form as

$$Z\,\bar{I} + Y = X \qquad (3.2)$$

where Z is matrix of intersector flows, $\bar{I}$ is a column vector of n rows each element of which is one, Y is a vector of final demands, and X is a vector of total outputs.

Equation 3.1 may be rewritten in terms of final demand as (3.3)

$$\begin{array}{lllllll}
X_1 - x_{11} - x_{12} & & - \cdots - x_{1j} & - \cdots - x_{1n} & & = & Y_1 \\
-x_{21} & + X_2 - x_{22} & - \cdots - x_{2j} & - \cdots - x_{2n} & & = & Y_2 \\
\vdots & \vdots & \vdots & \vdots & & & \vdots \\
-x_{n1} & - x_{n2} & - \cdots - x_{nj} & - \cdots + X_{nn} & - x_{nn} & = & Y_n
\end{array}$$

The system in 3.3 may be written in matrix notation as

$$X - Z\,\bar{I} = Y \qquad (3.4)$$

In 3.3 when i equals j is non zero, the total output is described as gross output and if i equals j is zero, the output is a net one.

The intersectoral flows, depicted either in physical or in value terms, describe the structural interdependence. Leontief, in his model, makes the simple assumption of fixed technical coefficients. The technical or the input-output coefficients a_{ij}'s denote the amount of output of the i th sector required to produce one unit of output of the j th sector. Assuming linear relationship between the purchases of the endogenous sectors and the total output, the input-output coefficient may be obtained as

$$a_{ij} = \frac{x_{ij}}{X_j} \tag{3.5}$$

where a_{ij} is the technical coefficient in the i th row and j th column, x_{ij} is the output of sector i used in sector j, and X_j is the total value of the j th producing sector's output.

From 3.5, it follows

$$x_{ij} = a_{ij} X_j \tag{3.6}$$

Equation 3.6 implies that the total amount of output used by sector j from sector i is equal to the amount of sector i output used per unit of output from sector j. Substituting this relationship in the system of Equations 3.3

and considering in net terms we get

$$
\begin{array}{l}
X_1 - a_{12}X_2 - \cdots - a_{1j}X_j - \cdots - a_{1n}X_n = Y_1 \\
-a_{21}X_1 + X_2 - \cdots - a_{2j}X_j - \cdots - a_{2n}X_n = Y_2 \\
\vdots \qquad \vdots \qquad \vdots \qquad \vdots \qquad \vdots \\
-a_{n1}X_1 - a_{n2}X_2 - \cdots - a_{nj}X_j - \cdots + X_n = Y_n
\end{array}
\tag{3.7}
$$

The system 3.7 may be expressed in matrix notation as

$$(I - A)X = Y \tag{3.8}$$

where A is the matrix of technical coefficients, X is the total outputs and Y is the vector of final demands.

Aitkin (1951) has shown that the system 3.7 of n linear equations with n unknowns can have a general solution only if the matrix of coefficients in the lefthand member is nonsingular. Hawkins and Simon (1949) have proved that the system of nonhomogeneous equations can have economic meaning only if X_i's are positive and a necessary and sufficient condition for all X_i's to be positive is that all the principal minors of matrix A be positive.

The final demand for the i th commodity Y_i is established by the relationship from 3.7 as

$$Y_i = X_i - \sum_{j=1}^{n} a_{ij} X_j \qquad (3.9)$$

where Y_i is the total autonomous demands from sector i and X_i is the net output of sector i, a_{ij} is the input-output coefficients and X_j is the total output of sector j.

Given the final bill of goods $Y_1, Y_2, \ldots, Y_n$ and the constant input-output relationship a_{ij}'s, the system 3.7 can be solved for the output levels $X_1, X_2, \ldots, X_n$ required to meet the specified final bill of goods. The solution is given by

$$X = (I - A)^{-1} Y \qquad (3.10)$$

or

$$X = BY \qquad (3.11)$$

where I is an identity matrix, A the matrix of input-output coefficients, Y is the final demand vector, X is the vector of outputs and B is the $(I - A)^{-1}$.

Expressing outputs as a function of final demands yields the system of equations 3.12 shown on page 63. The b_{ij}'s are known as the interdependence coefficients. Interdependence coefficients describe the required change in the (gross or) net output of industry i for a unit change in the amount of goods delivered to final demand by industry j.

There are several methods by which the matrix $(I - A)^{-1}$ may be computed. Heady (1958) suggests the Crout

$$
\begin{aligned}
b_{11}Y_1 + b_{12}Y_2 + \cdots + b_{1j}Y_j + \cdots + b_{1n}Y_n &= X_1 \\
b_{21}Y_1 + b_{22}Y_2 + \cdots + b_{2j}Y_j + \cdots + b_{2n}Y_n &= X_2 \\
\vdots \qquad \vdots \qquad\quad \vdots \qquad\quad \vdots \quad &\quad \vdots \\
b_{n1}Y_1 + b_{n2}Y_2 + \cdots + b_{nj}Y_j + \cdots + b_{nn}Y_n &= X_n
\end{aligned}
\tag{3.12}
$$

method, while others use the Doolittle method described by Anderson and Bancroft (1952). A simple method of power series expansion has been suggested by Waugh (1950) for calculating an approximate inverse. The progression

$$1 + a + a^2 + a^3 + a^4 + \ldots$$

has infinite number of terms, but if 'a' is less than 1, then the terms converge so fast that there is a limit to the value of progression. By adding more terms, one can make it closer to 1/1-a. Similarly, the matrix $(I - A)^{-1}$ may be calculated by the series expansion, if the sum of the numbers in each column is less than or equal to one. To check computational accuracy, one can compute the product $(I - A)(I - A)^{-1}$. If the computations are accurate, the product will be equal to the identity matrix or close to it. The errors by the method of series expansion have been found to be extremely small by Waugh (1950) even in the case of large sized matrices.

Price equilibrium

From Equation 3.6 the quantity of output needed from the first sector to support a given level of activity of the j th sector is given by a_{1j}. If this quantity is multiplied by p_1, the price of the first sector's output, we get the monetary value of output required from the first sector by the j th sector for its level of activity. This can be represented in equational form as

$$(a_{1j}X_j)p_1 + (a_{2j}X_j)p_2 + \dots + (a_{nj}X_j)p_n + V_j = X_jp_j \quad (3.13)$$

where a_{ij}'s are technical coefficients and p_i is the price of i th sector's output.

System 3.13 may be written in matrix form as

$$\bar{X}A'p + V = \bar{X}p \quad (3.14)$$

where $\bar{X}$ is the diagonal matrix of production levels, A' is the transpose of the matrix A, V is the column vector of payments to autonomous sectors and p is the column vector of prices for the intermediate sectors. The system 3.14 can be solved for prices as

$$p = \left[(I - A')^{-1}\right] \bar{X}^{-1} V \quad (3.15)$$

It can be seen from Equation 3.15 that the same inverse matrix that represents a general solution for production equilibrium becomes in its transpose form a solution to problems of price equilibrium. The column vector V may be comparable to the factor payment side of the gross national product.

The dynamic system

The incorporation of stocks and flows into the basic input-output system transforms it also into a dynamic theory, which refers not only to the inter-relationships between the different parts of the economic system at any given point of time, but also formulates its 'law of change' over time. In the original static equation, all additions to stocks have been treated as parts of the autonomous final demand. The dynamic formulation introduces another set of structural equations in which the stock S_{ij} is related to output X_j. The relationship may be expressed as

$$S_{ij} = b_{ij} X_j \qquad (j = 1, 2, \ldots, n) \qquad (3.16)$$

where the constant b_{ij} denotes the size of stock of commodities required per unit of the output of commodity j or it can be referred to as the capital coefficient of commodity i in industry j.

The rates of change in output $\dot{X}_j$ and the corresponding rate of change of stocks required to produce these outputs may be obtained by differentiating Equation 3.16 with respect to time as

$$\dot{S}_{ij} = b_{ij}\,\dot{X}_j \qquad (j = 1, 2, \ldots, n) \qquad (3.17)$$

The dynamic balance equation can now be written as

$$X_j - \sum_{j=1}^{n} a_{ij}\,X_j - \sum_{j=1}^{n} b_{ij}\,\dot{X}_j = Y_j \qquad (3.18)$$

$$(j = 1, 2, \ldots, n)$$

The dynamic equation differs from the static equation in that there is an additional term $b_{ij}\,\dot{X}_j$, which connotes the 'investment demand' for the products of industry i.

The solution of the system 3.18 of linear differential equations describes the 'law of growth' of individual outputs. The solution may be written as

$$X_j = \sum_{j=i}^{n} c_i\, e^{\lambda it} + L_i\,(Y_1, Y_2, \ldots, Y_n) \qquad (3.19)$$

where X_j is the output of the j th sector, c_i and λ_{it} are the coefficients and roots of the auxiliary equation and L_i is the linear function of final demands.

In numerical form, the solution aids the policy maker in explaining the behavior of the economic system over time. The solution further shows that the growth of outputs follows an exponential path.

A Numerical Example

This section provides a numerical illustration of the mathematical model described earlier. As detailed structural analysis of the Indian economy is described in the empirical section, a simple three sector model is used here for illustration.

A three sector transaction matrix

Table 9 is an oversimplified, aggregated version of the Indian economy for the year 1949-50 described by Uma Datta (1957). The economy is divided into four sectors *viz.*, agriculture, industry, services and households. Agriculture (X_1), industry (X_2) and services (X_3) are endogenous, while households (Y) are exogenous.

Row 1 in Table 9 shows that the gross output of agriculture was 58 thousand millions of rupees. The internal purchase by the agriculture sector was 9.77 thousand million rupees. The consumption of seeds for further production, feed for livestock etc. are some of the demands from its own sector. Row 1 Column 2 describes the purchases of industries from agriculture, which amounted to 8.32 thousand million rupees. Agricultural outputs, as cotton, oilseeds, etc. are purchased by the industrial sectors for their output. The purchases of the services sector from agriculture sector was 1.40 thousand million rupees and is shown in Row 1

Table 9. Transaction matrix (in thousand million rupees)

Producing sectors	Purchasing sectors: Agriculture 1	Industry 2	Services 3	Household 4	Total output
1. Agriculture	9.77	8.32	1.40	38.64	58.13
2. Industry	2.08	6.15	2.11	26.75	37.08
3. Services	0.77	5.99	2.47	29.54	38.74
Total input	12.62	20.46	5.98		
Total domestic product or value added	45.51	16.66	32.76	94.93	
Total product	58.13	37.08	38.74		133.95

Column 3. The households purchased from the agricultural sectors 38.64 thousand million worth of produce. The last column in Row 1 shows the total gross output of the agricultural sector. Similarly, the allocation of the outputs of industries and services are described in Rows 3 and 4 respectively.

As the rows describe the outputs of the sectors, the columns describe the cost structure of the different sectors. Column 1 describes the input of agriculture, industry and services in the agriculture sector. The total input in agriculture is shown at the bottom and it amounted to 12.62 thousand million rupees. Or in other words, with an input of 12.62 thousand millions, agricultural output was 68.13 thousand million rupees. The balance of 45.51 thousand million is the value added or the domestic product. In input-output terminology, it is the primary inputs, as labor costs, taxes, etc. In national income accounting, gross domestic product is treated as returns to factors. For example, the households provide labor for the different industrial activities and the government provides services. The returns to labor (factor) are in the form of wages and to the government returns are in the form of taxes. Columns 2 and 3 describe the cost structure of the industry and services sectors respectively. Column 4 shows the cost structure of the households. It will be seen from Table 9 that the total cost of households equals the value added or the returns to factors employed.

Input-output coefficients

The input-output coefficient a_{ij} describes the value of output of the i th producing sector required per rupee worth of output of the j th consuming sector. The input-output coefficients can be computed from Table 9 by applying the relationship given in Equation 3.5.

The matrix of input-output coefficients is

$$A = \begin{bmatrix} .1681 & .2244 & .0361 \\ .0358 & .1659 & .0545 \\ .0013 & .1605 & .0638 \end{bmatrix} \qquad (3.20)$$

The matrix of input coefficients is computed by dividing the elements in each of the columns in Table 9 with its corresponding gross output. For example, all the elements in Column 1 were divided by the total output for agriculture, the elements in Column 2 by the total output for industries and the elements in Column 3 by the total output for services. The element in Row 1 Column 1 in Equation 3.20 is .1681 and this is obtained as 9.77 / 58.13. Similarly, the element in Row 2 Column 3 is .0545 and is computed as 2.11 / 38.74. In the same manner all the input coefficients are calculated and presented as matrix A.

The input-output coefficients may be interpreted as the requirements per rupee worth of output. For one rupee worth of agricultural output .17 rupee of internal purchases .04 rupee worth of industrial purchases and .001 rupee worth services were required. The per unit cost structure of the industrial and services sectors are indicated in Columns 2 and 3 respectively of matrix A in System 3.20.

Interdependence coefficients

Earlier in the section interdependence coefficients b_{ij}'s were defined as the amount by which output of the i th producing sector will increase for each rupee increase in the final demand for the product of the j th sector. The matrix of interdependence coefficients (B) is obtained by computing the $(I - A)^{-1}$. The inverse for the matrix in System 3.20 is given below:

$$I - A^{-1} = B = \begin{bmatrix} 1.21683 & 0.34021 & 0.06666 \\ 0.05295 & 1.22723 & 0.07343 \\ 0.01072 & 0.21084 & 1.08084 \end{bmatrix} \qquad (3.21)$$

Expressing output levels as a function of final demands yields the relation similar to Equation 3.12. If we specify the final demands shown in Column 4 of Table 9, the output levels will be as shown in Column 5 of the table. The system may be written as (3.22)

$$X_1 = 1.21683x38.64 + .34021x26.75 + .06666x29.54 = 58.13$$

$$X_2 = .05295x38.64 + 1.22723x26.75 + .07343x29.54 = 37.08$$

$$X_3 = .01072x38.64 + .21084x26.75 + 1.08084x29.54 = 38.74$$

<u>Application of interdependence coefficients</u>

Let us consider the economy described in Table 9. Also assume that the planning authority in India estimates certain increases in the final uses, say, in a planning period of five years. The planners are now interested in knowing the output composition of different sectors required to support the increased final bill of goods. In such a situation the interdependence coefficients are useful.

For illustration, assume that the planners estimate the following increases in the final demands taking into account per capita income, population increases, foreign trade, et cetera:

1. Agriculture 10 percent
2. Industry 30 percent
3. Services 10 percent

The increased values of final products to be made available for household consumption computed from Table 9 are given below:

1. Agriculture 42.50 thousand crore of rupees
2. Industry 34.78 thousand crore of rupees
3. Services 32.49 thousand crore of rupees

The total outputs needed to support these increased final demands can be computed by applying the system of equations described in 3.12. The total outputs are calculated as (3.23)

$$X_1 = 1.21683 \times 42.50 + 0.34021 \times 34.78 + 0.06666 \times 32.49 = 65.71$$

$$X_2 = 0.05295 \times 42.50 + 1.22723 \times 34.78 + 0.07343 \times 32.49 = 47.32$$

$$X_3 = 0.01072 \times 42.50 + 0.21084 \times 34.78 + 1.08084 \times 32.49 = 42.91$$

From the system of Equations 3.23, the increased total gross outputs needed to support the increases in final demands are obtained. The new output of agriculture has to be 65.71 thousand million rupees, industry has to produce 47.32 thousand million rupees worth and services has to reach 42.91 thousand million rupees.

It will be interesting to know the extent of intermediate uses consequent on the increased total outputs, for a planner is primarily interested in knowing the capacity requirements and whether the available capacities can meet the increased outputs. The estimated or increased outputs, intermediate uses and total output are described in Table 10.

From Table 10, it will be seen that for the desired increase of 10 percent (percentages shown in parentheses) in final demands for agriculture, an increase of 19.09 percent or 23.21 thousand million rupee worth of intermediate products have to be produced. The estimated increase in total output of agriculture is 13.04 percent. In the case of industry, for an increase of 30 percent of final demand, an increase of 21.28 percent in the intermediate uses is required and 27.62 percent increase in the total output is necessary. For the increase of 10 percent in services sector, 14.25 percent increase in intermediate uses and 10.76 percent increase in total demand are needed. To meet the specified increases of final demand the total gross output will have to be increased

Table 10. Changes in total output and intermediate uses (in thousand million rupees)

Product originating from	Intermediate uses		Final demand		Total output	
	Given	Estimated	Given	Desired	Given	Estimated
1. Agriculture	19.49	23.21	38.64	42.50	58.13	65.71
		(19.09)[a]		(10.00)		(13.04)
2. Industry	10.34	12.54	26.75	34.78	37.08	47.32
		(21.28)		(30.00)		(27.62)
3. Services	9.12	10.42	29.54	32.49	38.74	42.91
		(14.25)		(10.00)		(10.76)
4. Total				133.95		155.94
						(16.42)

[a]Figures shown in parentheses are percentages.

by 16.42 percent. Thus, given the specified final demands, the interdependence coefficients are useful for finding out the extent of demands of intermediate sectors and also the total output levels required to support the desired final demands.

Assumptions of the Input-output Model

Several restrictive assumptions are made mainly to facilitate empirical applications. One of the important assumptions is that of constant returns to scale. It is often contended that functions are more complex than simple proportions in describing the production process. For example, in industries like railways and power at least one large installation must be provided before any output appears. But, the assumption of constant returns to scale is made for computational simplicity.

A second assumption is that there is no substitution among inputs. Gathering of data is simpler if an industry is regarded as a single process with fixed input coefficients. Samuelson (1957) has proved that absence of substitution need not be assumed in an open model, if there is just one scarce resource. Though the production functions allow substitution among inputs, it does not take place, no matter how the final bill of goods is changed, because the achievement of efficiency in production always leads to a unique set of input ratios for each industry. In actual situations, there is more than one scarce resource and hence, the assumption is being made again for simplicity and empirical expediency.

Another assumption is made about the absence of joint products, i.e., every process produces only one output. If a process produces two or more outputs in constant proportions, as hides and meat for example, then a single new output for the process could be defined which will meet the assumption. In contrast, in linear programming, it is taken that a process can produce several outputs.

An additional assumption is made in static models that only current flows of inputs and outputs are important. The problems of capacity and capital are not taken into account. In the dynamic models this aspect is taken care of by the inclusion of stock-flow coefficients.

Earlier in the section, we considered a four sector model in which the different industries of the Indian economy were aggregated into three sectors, besides the exogenous household sector. In the input-output analysis, the solution to any analysis depends on the nature of aggregation. Since aggregation of sectors is crucial to the model, we shall consider the different aspects of aggregation in the next section.

AGGREGATION AND SECTORAL CLASSIFICATION

The Walrasian general equilibrium does not consider aggregation of industries or products and one must, therefore, assume that each product and factor be assigned a set of equations. If, for example, there were 1000 products and 100 factors, there would be 2199 equations and unknowns requiring solution. Further, the classical general equilibrium theories were not concerned with the applications of their system for solutions to real problems. But, if one attempts to apply such system of equations, the magnitude of the number of equations and unknowns has to be within manageable proportions. Thus, one is left with one of two alternatives, either the data has to be aggregated, in which some information is bound to be lost, or to retain the detailed information at the expense of inability to solve the large system of equations.

Problem of Aggregation

In the macro economic models, considered earlier, relations are generally established among the few large aggregates, as national income, total investment, total consumption, et cetera of the economy. These aggregate models are completely devoid of sectoral details and consequently, no economically meaningful conclusions could be drawn from the heterogeneous aggregated data. In these models, the problem of aggregation is mostly ignored and the conclusions may, therefore, vary with changes in the composition of the aggregates.

The aggregation process props up in the input-output analysis essentially to reduce the number of equations and unknowns. Balderston and Whitin (1954) have observed that input-output model employing systems of simultaneous equations lends itself to the handling of more variables than multiple correlation analysis, thus facilitating the inclusion of detail on industry inputs and outputs. However, when simultaneous inter-relationships are taken into account, considerable detail is lost. Leontief (1951) recognized this fact and has stated that "the practical choice is not between aggregation and non-aggregation, but rather between a higher and lower degree of aggregation."

The problem of aggregation in macro models has been discussed extensively by Dresch (1938), Klein (1946), May (1947), Shou Shan Pu (1946) and Fisher (1958), to mention only a few. One of the criteria set down by Klein (1946) stipulates that if functional relations exist for input and output of an individual firm, there should also exist functional relations that connect aggregate output and input for the economy or parts of it. Similarly, the marginal productivities of the firm and the aggregated industry should be proportional under perfect competition. As the input-output model does not consider the marginal productivities, the second criterion is applicable only to general equilibrium situations of the Walrasian type.

Some of the difficulties that are encountered in the aggregation process of the input-output models are of a mathematical nature and others relate to the classification of industries and interpreting solutions. For the input-output model to be of practical use in planning economic programs, it is essential to formulate criteria based on sound theoretical principles for aggregating the available data.

In the input-output model, the total output of industry i can be expressed as

$$\sum_{j=1}^{n} B_{ij} Y_j = X_i \qquad (i = 1, 2, \ldots, n) \qquad (4.1)$$

where B is the $(I - A)^{-1}$ and Y_j is the final demand and X_i is the output.

Let us now assume that we aggregate some industries and denote the aggregate by I. The new sector may be written as

$$X_I = \sum_{i \in I} X_i \qquad (4.2)$$

where ϵ denotes "is an element of the set." The linear equations for the new system can be stated as

$$X_I - \sum_J x_{IJ} = Y_I \qquad (I = 1, 2, \ldots, m) \qquad (4.3)$$

or $$X_I - \sum_J a_{IJ} X_I = Y_I$$

Solving for aggregated outputs yields

$$\sum_J B_{IJ} Y_J = X_I \qquad (4.4)$$

where A_{IJ} is the amount of the I th industry's output required for a unit of final demand for good J.

Now assume some industries g, h in the first classification of n industries remain ungrouped in the second classification of m industries also. It will be possible to find the levels of output h for changes in the demand for g from Equations 4.1 and 4.4, and it can be seen that the solutions will differ in both the cases. Therefore, it is difficult to find out the type of aggregation that would provide the 'correct solution'.

As different methods of aggregation yield different solutions, a careful study of the objective of the model is necessary. An aggregation admirably suited for a particular study may be totally useless for another purpose. An input-output table should be detailed as far as possible. For example, an increased level of activity in iron and steel industry might necessitate increased production of coal. If coal is not shown separately in the input-output table, the difficulty of increasing coal proportionate to the steel production would be hidden in some aggregate fuel sector, which may even result in attempting an impossible attainment. Thus, aggregation often hides bottlenecks and shortages.

Some suggest that for purposes of aggregation, the various components to be aggregated should form a constant proportion of the aggregate. Proportionality of prices is not a sufficient condition for aggregation, unless outputs also vary proportionately. As demand elasticities are likely to vary, it is not likely that the total value of the products will also vary proportionately concomittant with proportionate price changes.

According to Samuelson (1948), a number of variables may be treated as a single variable when they are substitutable, i.e., when each has the same effect upon all the economic functions. Perfect substitution is rare, but by relaxation of rigid standards they can be approximated. Commodities which are close substitutes on the output side may not be substitutes in the input side also. Cotton and rayon are good substitutes from the consumers' point of view, but the raw materials making the inputs are altogether different. Normally, these two would be aggregated, but if, say, rayon is imported and if one of the aims of study is concerned with the nature of import, then it would be desirable to keep the sectors separate. As indicated earlier, one of the important aspects of aggregation is to ascertain in advance the uses to which the aggregated table is to be put and to align the data to meet the desired objectives.

Some commodities which are substitutes in certain activities may not at all be substitutes in other activities. Coal and oil may be substitutes in household, but for the production of steel, they are not.

Balderston and Whitin (1954) have clearly brought out the degree of feasibility of aggregated solutions.

> Assume that there exists perfect substitutability between components of each aggregate and also assume lack of substitutability between aggregates. In so far as the former assumption exagerates the degree of substitutability, solutions might be thought feasible which were not economically attainable. The latter assumption, in underestimating the degree of substitutability, may indicate that some solutions are not feasible which in fact are economically attainable. Thus, highly aggregative systems are likely to overestimate the attainable levels of production and consumption. On the other hand, systems involving much less aggregation are likely to underestimate the potentialities of the economy.

Another criterion for aggregation suggested by Samuelson (1948) is complementarity, i.e., commodities which have an invariant relationship to each other. Industries which are vertically integrated, as metal wiring, metal fabricating and metal products industries might be grouped into a single sector. But one difficulty in such a grouping is that prices are likely to fluctuate disproportionately and to carry it through the component parts of the aggregated sector is quite tedious. Hicks (1946) has stipulated the conditions under which the price movements may be in opposite directions in certain complementary goods. In this case, the total value components may not be proportional.

It is often suggested that aggregation of sectors with similar input coefficients could be effected. In order to ascertain the conditions under which two sectors could be grouped, let us consider the input coefficients of the aggregated sector. Denote the grouped sector as

$$X_{(m+n)} = X_m + X_n$$

$$a_{i(m+n)} = \frac{X_{i(m+n)}}{X_{(m+n)}} = \frac{X_{im} + X_{in}}{X_m + X_n}$$

$$= \frac{a_{im} X_m + a_{in} X_n}{X_m + X_n} = a_{im}\left(\frac{X_m}{X_m + X_n}\right) + a_{in}\left(\frac{X_n}{X_m + X_n}\right) \tag{4.5}$$

$$= w_m a_{im} + w_n a_{in}, \quad \text{where } w = \frac{X_m}{X_m + X_n}$$

Chenery and Clark (1959) have stated that if all the input coefficients $a_{i(m+n)}$ of the grouped sector are unaffected by changes in the output levels X_m and X_n, the demands of the consolidated sector for the output of other sectors will be equal to the sum of the demands of its components. If a_{im} equals a_{in}, i.e., input coefficients are similar, then no change in the weights resulting from changes in the proportions in which X_m and X_n are demanded will affect the aggregate coefficient.

The main aim of aggregation is to produce the minimum average error for all the production totals of the solution. When the objectives of the analysis is specified in advance, the importance of possible errors can be estimated better and the bases for aggregation could be adjusted according to the problem on hand.

Conceptual Background for the Indian Inter-industry Table

The picture of inter-industrial relationships may be considered as an extension of accounting of national product. The national product could be considered either in terms of market values (viz., supply prices of producers plus the indirect or commodity taxes net of subsidies) or in terms of supply or producers' prices. In an input-output table, the total delivery of a domestic sector should tally with some prior estimate reckoned at producers' prices. Imports have to be classified according to the flowing sectors and not in respect of the ultimate utilizers. As the input-output table constructed by the Indian Statistical Institutes for the year 1953-54 (Appendix A) is on the basis of market value, the appropriate duties also have to be deducted. The conceptual framework for the table provided by the Indian Statistical Institute (1957) can be schematically represented in accounting principles as

Product at market price = Product at supply price + taxes

= sum of deliveries + exports - imports

= all material inputs (domestic and foreign) + wages + rents + entrepreneurial earnings

An inter-industry table is similar to a double entry bookkeeping scheme applied to the several sectors of a nation's economy. The column of an activity gives the entries on the debit or cost side of ordinary accounts. In constructing a table, especially for less developed economies, the data problems are considerable and several approximations have to be made in the process of construction. The broad classification may be as producing sectors and consuming sectors. The sectoral classification and the problem of aggregation is crucial in an inter-industry table. As regards the consuming sectors, non-profit organizations like hospitals, educational institutions, and research institutions have not been included either in households or in government consumption, but they have been treated along with services. Defense capital outlay has been considered as an element of government consumption. Current outlays, as road maintenance, et cetera, though forming part of government consumption, cannot be normally separated out from the structure of the economy.

Fixed capital formation is also taken as a non-producing sector. It is difficult to distinguish between current, operating and resting accounts in a two way table and therefore, transactions in building capital have been included in the receiving sector.

The producing sectors or activities or businesses can be divided into any arbitrary variety of groups. The sectors are composed of a number of establishments and the nature of each establishment is determined by its 'characteristic output'. In practice, the nature of data and the general structural characteristics dictate the demarcation of the sectors. Theoretically, sound grouping of the sectors will be to aggregate sectors with identical cost structure or to group sectors with similar distribution of outputs between sectors. In the Indian table, the former criterion was the guiding factor.

Three kinds of production sectors have been distinguished in the table. They are (a) commodity producing sectors, (b) distributive trades (wholesale and retail) and (c) transport and services, including ownership and upkeep of residential property. Commodity producing sectors were again subdivided into primary activities, as (1) agriculture, plantations, animal husbandry and mining, (2) secondary production, like large scale factory production of producer and consumer goods and small scale domestic production. Maintenance work is allocated to the appropriate delivering sectors, i.e., entries in diagonal cells.

The distributive trade essentially includes warehousing and storage in the course of transport of produce. The railways also undertake a considerable storage, but this is merged with the railway activity and is excluded from trade items. In the table "sellers' values" have been adopted since it might be more suggestive of true economic relationships. The main reason is that trade margin on purchases by households is far bigger than on any other transaction that it can be reasonably assumed that they are proportionate. The difference between the sellers' price and buyers' price could be entered as a payment by the buying sector to the trade sector. An alternative method is to record separately buyers' quota of transport charges apart from "pure" trade margin and the transport charges do not enter in the cost of operation of trade except in so far as there is a transport margin in the material inputs used up in the current operation of trade. The second procedure has been adopted for this table.

Tertiary productive activities include (a) banking, insurance and cooperatives, (b) professions, services and non-profit institutions and (c) upkeep of residential house property. Professions, however, include self-employed persons. The payments passing into the hands of individuals are recorded as wages.

As the inter-industry table is to analyze true economic relationships in real terms, only three kinds of transactions have been considered. These are (a) barters, (b) transactions compensated by money, i.e., buying and selling, and (c) real transfers, i.e., transactions not compensated. In as much as the main bulk of the transactions is of the second type, the table may be construed to represent the total financial transactions of the community. Gifts, government welfare measures, grants to institutions, et cetera are considered to pass directly to consumers.

Income has been regarded as payments to households and enterprises. Payments to government by way of taxes have been excluded because they are not relevant for industrial analysis. Subsidies have been treated as reduction in operating costs. All interest and premiums on insurance have been recorded as payments to the respective banking and insurance sectors as if they were real transactions. This is a modification of the table for 1950-51 prepared by Goodwin (1953), where interest and profit are treated as operating surplus. Profits are difficult to identify and they have been entered in the same manner as bookkeeping accounts, viz. as an item which closes the operating account.

In the table, distinction is made between current and capital accounts. First, for each sector, a current and capital account was drawn taking into account the real transactions. Then, all deliveries on capital account originating the capital or current accounts were merged in order to obtain the deliveries of producing sectors. However, capital accounts were not set up for households. The amount of goods which are regarded as capital represents a flow to the current and capital accounts of different productive sectors as well as of government. Flow of scraps is treated as a negative flow of output, but detailed information on this account was not available for a number of industries.

The nature of depreciation raises an important issue in the table. In as much as the table was not intended to depict dynamic aspects of the economy, capital formation was not shown as an activity and therefore, depreciation could not be shown as a delivery to the productive sectors. Instead, the practical approach of the National Income Committee, viz., entering depreciation as a current cost of maintenance at a discounted rate was adopted. Depreciation has been accounted as an item of input in the table and has been merged with material costs. Similarly, the estimate of capital formation is also net, i.e., new over and above depreciation.

Sources and Estimation Procedures

Most of the information for the table was collected from mainly three sources: (1) The National Sample Survey (1952) has conducted a number of rounds on agricultural and industrial production. The data from fourth and fifth rounds were adopted for the table. For the industries, the Sample Survey of Manufacturing Industries and the detailed Census of Manufacturers on Indian Manufacturing Industries were available. (2) Considerable data was also culled out from a number of government publications of the state and central governments and also from the publications of Reserve Bank of India. (3) Use was also made of the unpublished dossiers of the Central Statistical Organization for verifying allocation estimates.

In constructing the table, account had to be taken of the nature of estimates available from the National Sample Surveys. The surveys indicated the payments handed out by different types of activity. By allocating all the outlays for the different recipient sectors, we also obtain deliveries by the sectors. The expenditure end and allocation of deliveries had to be tallied at different stages for purposes of confirmation.

The national estimates available from the National Sample survey yield information only on heavily grouped inputs and therefore, they had to be adopted for equally heavily aggregated sectors. As a result, the outlays of productive sectors, as small scale and cottage production, could not be accurately identified. Consequently, the table does not reveal linking of small scale and cottage production. It is also hard to guess from aggregate estimates of outlays as to how much represented a draft from large scale enterprises and how much from the corresponding small scale industries. The ratio of the value of output to the value added may be a useful guide, but has to be supplemented in each case by sound judgment and other outside information, emanating from the nature of linkage of the major industry to the small scale sectors.

The estimates in the autonomous sectors contain consumption, private and public investment and net exports. Household consumption, by groups of items, is available in the surveys of the National Sample Survey, but since they are heavily lumped in final estimates, it was estimated as a residual. Fairly reliable estimates are available for outlays of government and local authorities and they are assumed as given. Exports and imports are ascertained from the Annual account of Sea and Airborne Trade of India.

Transport charges have been deducted from the different sectors and shown separately against the corresponding sectors. Similarly, duties and taxes were separated out and entered under the column representing the group of products. These were then checked with the net value added and the total value product of the sectors.

EMPIRICAL APPLICATIONS

In this section, an empirical analysis of the Indian economy is presented. In the investigations, the 1960-61 projection of the inter-industry table published by the Indian Institute of Public Opinion (*ca*. 1960) has been used. This table provides considerable detail about the agricultural sectors, as wheat and other cereals, rice, industrial crops, livestock products, et cetera. The table is relatively small, the total economy being divided into nineteen sectors. Another advantage is that the table is in producers' prices and comparisons with other countries is, therefore, possible.

Structural Analysis

The table (Appendix B) may be broadly divided into two subdivisions, *viz*., agriculture and industry. The first five sectors represent agriculture and the rest are industrial sectors. The nineteen sectors are identified as follows:

1. Wheat and other cereals
2. Rice
3. Industrial crops
4. Livestock and products
5. Other industrial crops
6. Food manufactures
7. Vegetable oil

8. Wood
9. Jute
10. Textiles
11. Leather and rubber
12. Fuel and power
13. Mining
14. Basic metal industries
15. Metal products
16. Non-metallic products
17. Chemicals
18. Construction
19. Transport

The nineteen sector table also provides data on exogenous final demands, as exports, government expenditure, investment and consumption. As the table describes the transactions at producers' prices, trade margins are excluded from the flows depicted in the table and is considered as exogenous input.

Let us now consider the analytical uses of the inter-industry table. To ascertain the extent of inter-industry current uses and the final uses of the gross total output, the percentages for both these uses out of the total gross output were computed. The allocations are presented in Table 11.

Table 11. Allocation of total output for inter-industry and final uses (in percentages)

	Sector	Total inter-industry uses	Final uses
1.	Wheat and other cereals	6.95	93.05
2.	Rice	9.98	90.02
3.	Industrial crops	86.74	13.26
4.	Livestock and products	28.90	71.09
5.	Other industrial crops	35.62	64.38
6.	Food manufactures	6.07	93.93
7.	Vegetable oil	11.41	88.59
8.	Wood	70.24	29.76
9.	Jute	32.91	67.08
10.	Textiles	11.33	88.67
11.	Leather and rubber	5.05	94.95
12.	Fuel and power	18.49	81.51
13.	Mining	30.58	69.42
14.	Basic metals	84.76	15.24
15.	Metal products	30.72	69.28
16.	Non-metallic products	92.02	7.99
17.	Chemicals	47.12	52.88
18.	Construction	4.12	95.88
19.	Transport	60.37	39.63

From Table 11, it would be seen that in eight of the industries, more than eighty percent of the gross total output moves directly for final uses and less than twenty percent is demanded for inter-industry uses. The few sectors in which inter-industry uses are of considerable magnitude are industrial crops, wood, basic metal industries, non-metallic products and transport. These sectors, as one would expect, supply their outputs as inputs for other industrial sectors. In all the other cases, products moving directly for final uses exceed sixty percent of gross output. This implies that in a less developed economy, the network of processing industries is not as complicated as in developed economies and there is less dependence of one industry for its input on the output of another industry. Most of the goods do not obviously undergo elaborate stages of processing. It might be noted that in the case of the agricultural Sectors 1 to 5, except in Sector 3, viz., industrial crops, 64 to 93 percent move directly for final uses. Similarly, the final uses for food manufactures, vegetable oil, textiles, fuel and power exceed 80 percent of the total output.

The interdependence among productive sectors may also be investigated from the proportion of the factors of production employed in the processes of production of the commodity. The quantum of indirect use of factors can be measured by the ratio of purchased inputs to the total gross output of that sector. These ratios, calculated as percentages, are presented in Table 12.

It would be seen from Table 12 that the purchased inputs vary from 11 percent to 71 percent of the gross output of that sector. In the case of the first five agricultural sectors, excepting the livestock products, the ratio is low, varying from 11 to 16 percent. Similarly, in the case of wood and mining, the ratio is relatively lower than other industrial sectors. A high ratio of purchased input of 71 percent is noticed in sectors, like vegetable oils and non-metal products. Food manufactures is another sector which has purchased inputs amounting to 60 percent.

Let us now denote by w_i the ratio of intermediate total demand to the total output of the sector and by u_i the ratio of purchased input to output. These ratios are now used to classify the different sectors of the Inter-industry Table for India, depending on whether they are high or low. (See Table 13.)

In Table 13, we have adopted the classification of the types of productive sectors similar to the one followed by Chenery and Watanabe (1958) so that comparison of the Indian table could be made with those of Japan, United States and Italy. It is interesting to note that the classification of sectors for India follows more or less a similar pattern, though there are certain deviations. The figures of Chenery

Table 12. The ratio of purchased input to output

Sector	Percent
1. Wheat and other cereals	16
2. Rice	11
3. Industrial crops	12
4. Livestock and products	40
5. Other industrial crops	12
6. Food manufactures	60
7. Vegetable oil	71
8. Wood	25
9. Jute	68
10. Textiles	42
11. Leather and rubber	47
12. Fuel and power	33
13. Mining	22
14. Basic metals	44
15. Metal products	29
16. Non-metallic products	71
17. Chemicals	47
18. Construction	40
19. Transport	38

Table 13. Types of productive sector of India--1961-62

	By use of output: Final (low w)				By use of output: Intermediate (high w)			
By type of input	III.	Final manufacture	w	u	II.	Intermediate manufacture	w	u
	4.	Livestock products	.29	.40	14.	Basic metals	.84	.44
	6.	Food manufacture	.06	.60	16.	Non-metallic products	.92	.71
Manufacturing (high u)	7.	Vegetable oil	.11	.71	17.	Chemicals	.47	.47
	9.	Jute	.33	.68	19.	Transport	.60	.38
	10.	Textiles	.11	.42				
	11.	Leather and rubber	.05	.47				
	18.	Construction	.05	.40				
	IV.	Final primary production			I.	Intermediate primary production		
	1.	Wheat and other cereals	.07	.16	3.	Industrial crops	.87	.12
	2.	Rice	.09	.11	8.	Wood	.70	.25
Primary production (low u)	5.	Other industrial crops	.36	.12				
	12.	Fuel and power	.18	.33				
	13.	Mining	.31	.22				
	15.	Metal products	.29	.31				

and Clark (Table 8.2, 1959) are the average values of u and w for Italy, Japan and United States, and their table also distinguished 29 sectors, while there are only 19 sectors in our table. Food manufactures, vegetable oil, textiles and leather and rubber and livestock products (processed foods) get classified under final manufacture in both the cases, which have a low w and high u. Similarly, basic metals, as iron and steel, non-metals and chemicals fall under the grouping intermediate manufacture, with high w and high u values. In the table for India, services are considered as autonomous inputs and therefore, do not get classified under primary production. However, extractive industries, as agriculture and mining and fuel and power get classified as final primary production. The main difference is, however, in the transport sector, which falls under final primary production in the case of Italy, Japan and U.S., while in India it is under intermediate manufacture, and this is probably due to the differences in the industries going into aggregation. In India, ship building and railways are included in transport sector, while they are separate sectors in the tables of Chenery and Watanabe. Likewise, mining is classified under intermediate primary production for the three countries, while for India it comes under final primary production. This classification is similar to the classification of Colin Clark (1951). Colin Clark's classification of primary and tertiary industries are identified by the low values of w's and secondary production by high values of u's.

The classification of sectors in Table 13 brings out the structural interdependence in the process of production. The sectors under Category I, viz., intermediate primary production, have high intermediate demand and low purchased inputs. Category II, on the other hand, is characterized by high intermediate demand as well as high purchased inputs. The sectors falling under Category III, viz., final manufacture, have low intermediate demand and require high purchased inputs. For the final primary production sectors (Category IV) intermediate demand as well as purchased inputs are low.

Let us now consider some of the simple applications of the open input-output table. The activity levels can be expressed as a linear function of the final bill of goods, which can be written as

$$X = (I - A)^{-1} Y \tag{5.1}$$

where X is the vector of outputs, $(I - A)^{-1}$ is the inverse of the coefficient matrix and Y is the vector of final demands. The output of the i th sector X_i is, therefore, a function of all the final demands $Y_1, \ldots, Y_n$.

Before we investigate the levels of outputs required to meet given final demands, let us first consider the interpretation of the row and column sums of the inverse matrix. Appendix C shows the (I - A) inverse for the 19 sector model of the Indian economy for the year 1960-61. The sum of the rows of an inverse matrix indicates the rupee or dollar worth of output of each sector that would support a rupee or a dollar worth of final demand from each of the sectors. Or in other words, the final demand vector (Y) is a sum vector, sum vector being defined as the sum of n independent unit vectors if there are n sectors. The column sums of the inverse matrix have a different interpretation. The sum of the first column, for example, denotes the rupee worth of outputs of the different sectors that would be required to support only 1 rupee worth of final demand for Sector 1, i.e. for supporting a unit vector of final demand of the sector in question. The row and column sums of the inverse matrix are presented in Table 14.

The ranking of the sums of rows denotes the different magnitudes of the output of the sectors required to support one unit each of the final bill of goods. From Table 14, it would be seen that the output of industrial crops will have to be at its maximum for supporting a unit bill of goods of all the sectors. Next in the order of ranking is the transport sector and then other agricultural products and so on. In a similar investigation by Chakravarthi (1961)

Table 14. Row and column sums of the 19 x 19 inverse matrix for India, 1960-61

Sector	Sum of row	Rank	Sum of column	Rank
1. Wheat and other cereals	1.1083	17	1.2052	16
2. Rice	1.2614	11	1.1448	19
3. Industrial crops	3.1498	1	1.1647	17
4. Livestock and products	1.8610	4	1.4941	11
5. Other industrial crops	2.3571	3	1.1615	18
6. Food manufactures	1.0676	18	1.8067	4
7. Vegetable oil	1.2046	12	1.8507	2
8. Wood	1.4644	7	1.3734	14
9. Jute	1.1715	14	1.8304	3
10. Textiles	1.1296	16	1.5704	9
11. Leather and rubber	1.0433	19	1.7062	6
12. Fuel and power	1.8361	5	1.4586	13
13. Mining	1.3683	9	1.3292	15
14. Basic metal industries	1.4020	8	1.6302	8
15. Metal products	1.7849	6	1.4602	12
16. Non-metal products	1.1959	13	2.0925	1
17. Chemicals	1.2955	10	1.7590	5
18. Construction	1.1485	15	1.6867	7
19. Transport	2.3817	2	1.5074	10

for the 36 sector model for India for 1953-54, the ranking of distribution and trade was highest followed by agriculture and then livestock products and so on. In as much as the sectors in the 19 x 19 table are differently aggregated, the results are not strictly comparable. But, however, the ranking of the sectors follows a more or less similar pattern, though in some sectors there is considerable variation, probably due to the differences in the nature of aggregation of industries. This also, in a way, lends support to Leontief's (1951) hypothesis that different aggregations of sectors would lead to different solutions. In Table 14, the row sums of leather and rubber and food manufactures rank 19 and 18, respectively, which implies that their output has to be supported least for the unit demand of each of the final bill of goods.

Similarly, the column sums in Table 14 indicate the magnitude of a unit final demand (unit vector of final demand) of the different sectors of the economy on the total value of outputs of all the sectors. In other words, for supporting 1 crore (10 million) of final demand of non-metal products, which incidentally ranks 1, an output of 2.0925 crores of rupees worth of all the sectors has to be produced. Agricultural sectors, as wheat and other cereals, rice, industrial crops and other agricultural products occupy the lower rung of the rankings, which implies that for a crore of rupees worth of final bill of goods of each of these sectors, the output to be supported by all the sectors is relatively less. In the investigations of Chakravarthi (1961) also, the agricultural sectors are way down in the rankings.

The inverse matrix $(I - A)^{-1}$ is useful for investigating the output-composition of the economy, given the final bill of goods. In the inter-industry table for India (Appendix B), the columns 20 to 23 describe the final demands, *viz.*, exports, government expenditure, investments and consumption of the households. To obtain the output levels required to support each of the components of final demands, we have computed the output-composition for the different final demand vectors, as exports, government and private consumption and investment separately by applying Equation 3.12. The total disaggregated outputs to meet the different components of final demand are presented in Table 15.

It would be seen from the Inter-industry Table (Appendix B) that the government consumption is quite small and hence, it was added to the vector of private consumption for computing the output levels. Columns 1, 2 and 3 describe the output-composition of the 19 sectors needed to support the levels of final demands of exports, investment, government and private consumption respectively. Column 4 of Table 15 indicates the aggregate outputs needed to meet all the three

Table 15. Outputs required to meet the different components of final demand (in crores of rupees)

Sectors	Supply of output to meet demand of Exports	Investment	Govt. and private consumption	Total gross output
	1	2	3	4
1. Wheat and other cereals	1.3704	------	2213.3764	2214.7468
2. Rice	4.4278	5.5519	2088.6463	2098.6260
3. Industrial crops	186.1389	5.8842	878.8596	1070.8827
4. Livestock and products	43.0963	8.7104	1707.0942	1758.9009
5. Other agricultural products	283.6156	13.5297	2797.5646	3094.7099
6. Food manufactures	23.5528	0.1326	629.6622	653.3476
7. Vegetable oils	35.9008	2.5987	476.0003	514.4998
8. Wood	14.2853	53.4150	139.0542	206.7545
9. Jute	139.5503	6.8911	61.5351	207.9765
10. Textiles	158.0312	0.0013	927.8862	1085.9187
11. Leather and leather goods	35.5939	2.2618	124.6021	162.4578
12. Fuel and power	31.1129	57.1090	908.9507	997.1726
13. Mining	58.7363	29.6590	77.4613	165.8566
14. Basic metal industries	24.0711	88.5027	77.8941	190.4679
15. Metal products	39.9007	382.8979	316.0617	738.8603
16. Non-metal products	5.8180	80.2458	12.2018	98.2656
17. Chemicals	11.8515	18.0132	112.4834	142.3481
18. Construction	3.0593	506.8803	41.3496	551.2892
19. Transport	42.3851	53.3261	590.4319	686.1431

components of final demands. These structural details aid the planner to estimate the capacity requirements of individual sectors to meet a set of final demands, which are estimated from extraneous sources, as consumption patterns, change of tastes, population growth, availability of foreign and total investments, prospects of foreign trade, et cetera.

The imports are shown as an autonomous input item under Row 23 of the transactions matrix (Appendix B). Many of the developing countries are interested in replacing the imported goods with products of indigenous origin. Though a sophisticated study on imports would be to consider competitive and non-competitive imports separately, as has been done for the several Latin American countries by the Economic Commission for Latin America (1956, 1957, 1958), we are not in a position to undertake such an investigation for want of data separately under the two types of imports. However, such an approach is possible, if the imports could be separated out as competitive and non-competitive from export figures available in meticulous detail in the annual account of Sea and Airborne Trade of India. Within the limitations of available data, we will make the restrictive assumption that all the items of imports can be substituted by products of national origin and that they could be produced within the available technological process of production. Since our purpose is merely to estimate changes on the output-composition in case the imports are to be substituted, we have computed the output levels of the nineteen sectors required to replace imports. These are presented in Table 16.

So far, we have considered the output-composition required to support the different components of final demand and exports. Let us now investigate the effects of autonomous inputs, like exports, the cost of trade and services and the labor costs. The autonomous inputs are similar to the concept of 'value added' in national income accounting. The value of primary inputs is the difference between the value of production in a sector and the cost of inputs from productive sectors. Wherever the imports are treated as primary inputs, the column totals have to be considered as total supply rather than total domestic production.

In the input-output table for India for 1960-61 (Appendix B) four types of primary inputs have been distinguished. They are imports, trade and services, government expenditure and wages. As the wages in the table include returns to labor and also probably other factor incomes as rents, interests and profits, any refined analysis of employment is not feasible. In addition, wages do not account for incomes of self-employed persons and such incomes are likely to be high in less developed economies, particularly in agriculture. Therefore, our analysis of wages will reflect only wage earners and will not have any relation to the total employ-

Table 16. Outputs needed to substitute imports

Sectors	Output levels needed to replace imports	Imports during 1960-61
1. Wheat and other cereals	151.84	142.53
2. Rice	66.97	51.75
3. Industrial crops	149.54	118.75
4. Livestock and products	32.66	4.05
5. Other agricultural products	57.05	21.65
6. Food manufactures	16.42	15.22
7. Vegetable oils	10.55	4.68
8. Wood	38.07	25.58
9. Jute	2.08	0.02
10. Textiles	35.41	31.42
11. Leather and leather goods	3.32	3.01
12. Fuel and power	130.20	102.46
13. Mining	20.72	1.58
14. Basic metal industries	118.20	35.34
15. Metal products	437.24	395.85
16. Non-metal products	13.94	11.89
17. Chemicals	39.44	29.15
18. Construction	3.14	-----
19. Transport	59.55	-----

ment in the country.

The amount of labor inputs, *viz*., wages, imports and trade margins per unit value of output have been presented in Table 17. Government revenue has not been considered separately, for it is a fluctuating one depending on the policy of the government.

Table 17 describes the cost of the primary or exogenous inputs per unit value of output. It would be seen from the table that in the case of basic metal industries, per rupee worth of output, .54 rupee worth of imported inputs is necessary. Similarly, in chemicals, .20 unit of imported inputs is used for every unit of output. For expanding these sectors one must take into account the balance of trade position and the availability of foreign exchange. It should be the aim of the planners to analyze the various components of imported inputs and efforts should be made to substitute them gradually with products of indigenous origin. It is also seen that imported inputs are the least for jute per unit value of output.

A look into the trade costs would reveal that the input on this item is high in the case of wheat and other cereals, rice, mining and other agricultural products. In the case of agricultural products, it is known that the trade costs are high since the produce passes through a number of intermediaries before reaching the ultimate consumer. The

Table 17. Wages, trade costs and imports per unit value of output

Sectors	Imports	Trade costs	Wages	Gross value of primary inputs
1	2	3	4	5
1. Wheat and other cereals	.0643	.4033	.3052	.8418
2. Rice	.0247	.2234	.6309	.8890
3. Industrial crops	.1107	.0077	.7257	.8782
4. Livestock and products	.0023	.0734	.4982	.6011
5. Other agricultural products	.0070	.1988	.6313	.8767
6. Food manufactures	.0233	.1182	.1608	.3980
7. Vegetable oils	.0091	.0766	.1868	.2858
8. Wood	.1236	.0295	.5139	.7489
9. Jute	.0001	.0172	.1784	.3208
10. Textiles	.0289	.1299	.3459	.5757
11. Leather and leather goods	.0185	.0657	.3386	.5343
12. Fuel and power	.1028	.1384	.3297	.6722
13. Mining	.0095	.2719	.4280	.7787
14. Basic metal industries	.1855	----	.2782	.5625
15. Metal products	.5376	.0560	.0142	.7104
16. Non-metal products	.1207	.0011	.1497	.2866
17. Chemicals	.2040	.0363	.1593	.5293
18. Construction	----	.1671	.4176	.5957
19. Transport	----	----	.5298	.6089

input coefficients of trade and services guide the planners in undertaking marketing facilities so as to reduce the cost to the minimum.

Column 4 of Table 17 describes the input cost on account of wages. The input of labor is uniformly high in all the agricultural and allied industries. In the first five agricultural sectors, labor inputs range from 30 to 72 percent. Again in sectors like wood and mining, the labor inputs are high. Purely industrial sectors, as vegetable oils, food manufactures, jute industry, metal products and non-metal products and chemicals have low inputs of labor per unit value of output. These low labor input coefficients imply that the technology in these sectors is well advanced and the sectors are capital intensive and mechanized. Construction and transport have a high input coefficient and these larger inputs are partly due to deliberate planning to provide employment for the large army of unemployed. The relatively high input of labor in the textiles is probably due to the existence of a large handloom sector side by side with the mechanized textile mills. The leather and its products are, similarly, carried on more as a cottage industry than as an organized industrial sector. The availability of cheap labor to an extent accounts for the high input of labor in the different sectors.

Column 5 of Table 17 depicts the total primary inputs per unit value of output. In the first five agricultural sectors, excepting for livestock products, the total cost of primary inputs varies from 84 to 89 percent. Likewise, the cost of primary inputs exceeds 70 percent in the case of wood, mining and metal products. The high coefficient in the purely industrial sector of metal products is due to the large imported inputs. An interesting feature is that except in the case of wood, the primary inputs exceed 50 percent and therefore, the purchased inputs are less than fifty percent. This implies that most of the sectors depend heavily on exogenous sectors for inputs and the interdependence between the producing sectors is quite low.

So far, we have considered some of the structural aspects of the Indian economy. We now proceed to investigate the programming applications of the input-output system. Before casting the input-output model into the programming framework, it would be appropriate to first discuss the logic of linear programming and then to consider the generalized activity models.

Programming Applications

Positive and normative approaches

Before discussing the programming formulations, we shall consider the empirical tools available to economists and their conceptual approaches. There are two separate approaches commonly known as positive and normative analysis. Positive analysis has been described by Heady (1961) to mean prediction of quantitative relationships among variables as they actually do exist at a point in time or have existed over a period of time. This is analogous to saying that the models are descriptive or predictive. As the analysis describes the structure as it actually exists, it can be used to predict the magnitude of one variable from the magnitudes of others. On the other hand, normative analysis refers to what ought to exist under the assumptions made. If, for example, the planners of India have certain goals and knowledge, a normative analysis would indicate as to what would happen or is expected to take place under certain given conditions. One advantage of normative approach is that we could take into account certain new variables, which are likely to exist, into the programming formulation, while in positive analysis, we essentially use historic data of the past to predict the future. In essence, positive analysis is the extrapolation of past trends to predict the future events, and therefore cannot take care of any new variables that are likely to prop up in the planning horizon.

The major tools for the positive analysis are the different regression methods and the more recent input-output technique. The important techniques for normative analysis are budgeting and programming methods. In any planning model, our interest lies in the future course of events and not in the historic past. As a matter of fact large scale planning is resorted to to change the course of events, as increasing the supply of outputs, changing the production structure and technology. Regression models based on time series observations cannot tackle such problems. As such, one has to resort to programming techniques in order to analyze the possible effects of new variables and more detailed examination of specific variables.

Normative analysis has also certain inherent deficiencies. While global coefficients or regional aggregates can be handled readily by regression models, programming models built on national aggregates may have little meaning. Heady (1961) has observed that even if sufficient restrictions are introduced, the model could tell only the historic story and would have the same limitations as the regression models in predicting a future, subject to technological and institutional changes.

While both normative and positive approaches have their merits and demerits yet they have helped in analyzing factor demand and product supply in agriculture. Our empirical programming formulation of the input-output model presented later in the section is in a way blending of the positive and normative approaches.

Input-output analysis and linear programming

The concept of activity analysis has been discussed by Heady (1958), Koopmans (1957) and Chenery and Clark (1959). Activity analysis is a method of analyzing economic changes through smaller units known as activities. Activity analysis is more general than linear programming in that it covers a broad range of problems, as the classical theory of production and of the relation between production and prices, besides linear programming. While activity analysis just provides a set of concepts and their implications, linear programming may be considered as an empirical, mathematical formulation which provides numerical solutions under certain assumptions. Linear programming was initially developed to allocate resources or for working out production plans in a manner that would yield maximum profits or outputs or that would minimize cost. The technique has so much developed that it is now applied to a wide range of allocation and price problems.

Assumptions

(1) In both the input-output analysis and linear programming activities have to be additive. That is, the total amount of resources used by several enterprises must equal the sum of the resources used by individual enterprise. There cannot be any interaction in the amount of resources used whether the activities are produced alone or in varying proportions. We also assume lack of economies or diseconomies

and constant returns to scale in both the models. (2) The input-output model assumes that each commodity is produced by a single sector of production, while in linear programming, we take it that a commodity may be produced by several activities and each activity has several outputs. (3) In both the models, a linear homogeneous function is assumed (i.e., $a_{ij} = x_{ij}/X_j$). For linear programming, proportionality assumption is essential, while it is not important in the input-output model. (4) Non-negativity of activity levels is necessary in both the models. In the linear programming model a specific condition must be imposed, while in the Leontief's input-output model, it is a necessary property and is, therefore, built in the model itself. (5) Both linear programming and static input-output models employ the assumption that input-output coefficients, prices and resource supplies are known with certainty at a particular point of time.

Computational and conceptual similarities

Linear programming is applied to select the best possible choice or optimum program, while in input-output system, the solution revolves around determing the quantitative manner in which one sector is related to another. Heady (1958) has shown that in the simplex table, each of the input-output coefficients computed in the feasible solution of linear programming is also an interdependence coefficient. Thus, both the models have some close computational and conceptual similarities in determining solutions. If we denote by r_{ij} the input-output coefficients computed in each feasible solution in the simplex table, then r_{ij} indicates the amount by which the activity in the i th row will be increased or decreased per unit change of activity of the j th column.

Basic concepts of linear programming

In this part, we will discuss some of the programming concepts as resource restrictions, production possibilities and the optimum plan. These concepts have been discussed in detail by Heady (1958), Dorfman, Samuelson and Solow (1958) and by a number of others.

Resource restrictions

Table 18 contains the data for a simple two resource, two activity programming problem. For the purpose of illustration, we will take the agricultural farm planning problem provided by Candler (1957). In this example, there are two activities, potato and corn production (P_1 and P_2). These activities are limited by supply of land (P_3) and capital (P_4). For producing one unit of potatoes, 1 acre of land, 0.887 (hundred dollars) of capital are required and the net revenue is 0.834 (hundred dollars). Similarly, for producing one unit of corn, 1 acre of land, 0.350 of capital are needed and the activity yields a net revenue of 0.724 dollars. Production of potatoes and corn is limited by the farmer's limited supply of land and capital. The programming problem is to combine the production of potatoes and corn in such a way that the profit is maximized within the limitations of the supply of land and capital.

The problem described in Table 18 may also be geometrically expressed as in Figure 1. The vertical axis measures the potatoes in acres and the horizontal axis measures the corn in acres. The line r r' will yield a revenue of $1000 i.e., all combinations of the areas under potatoes and corn on the isorevenue curve r r' will yield a revenue of $1000. As we move away from the origin, we come across successively higher isorevenue curves. The profit or revenue is the maximum where the production possibility curve intersects the highest isorevenue curve. In Figure 1, the production possibility curve b c a intersects the highest isorevenue curve at the point c. So the optimum plan is 5.59 acres of potatoes and 54.41 acres of corn. The point c can also be shown to be the optimum plan by calculating the marginal rates of substitution and substituting in the criterion equation.

From Figure 1, it would also be seen that the feasible area or the production possibility open to the farm is bounded by the broken line b c a. Above b c, the limitation of capital does not allow for the full utilization of land and above c a, the limitation of land does not allow for the full

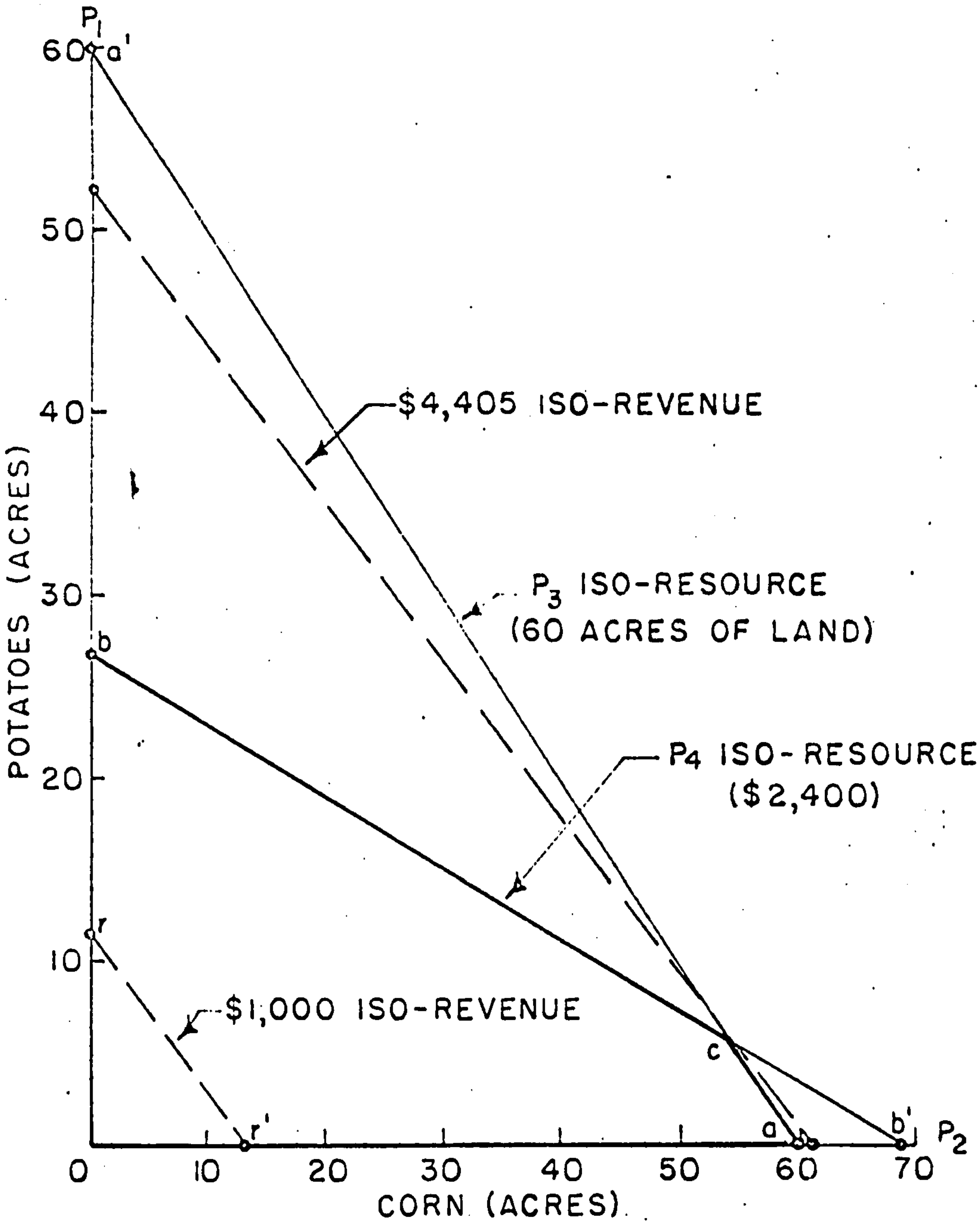

Figure 1. Production problem expressed in terms of acres of crop production

Table 18. Per acre requirements and resource supplies

Resource	Units	Activity		Supply
		Potatoes P_1	Corn P_2	P_0
Land P_3	Acre	1	1	60
Capital P_4	\$100	.887	.350	24
Net revenue	\$100	.834	.724	

utilization of capital resource. The distance between a' c and b' c indicates the amount of land that has to go unused for plans along the segment a' c and similarly, the amount of capital that has to go unused for points on b' c is given by the distance between c a and c b'.

In this simple case chosen for illustration, only a single program at c exhaust the resources, *viz.*, capital and land. If we allow for the nonuse of resources (or provide for disposals), there may be a more profitable optimal program in which some resources may go unused. In a problem, where there are a large number of activities and resource restrictions, an optimal plan that yields maximum profits may be one in which one or more resources may not be completely used up.

Production possibility curve

From Table 18, the production possibilities may be represented as

$$1p + 1c = 60 \qquad (5.2)$$

$$.887p + .350c = 24 \qquad (5.3)$$

Expressing the production possibility of potatoes as a function of corn we have

$$p = 60 - 1c \qquad (5.4)$$

$$p = 27.05 - .39c \qquad (5.5)$$

The coefficient 1 for corn is the ratio of

$$\frac{\text{land requirement per unit of corn}}{\text{land requirement per unit of potatoes}} = \frac{\Delta p}{\Delta c}$$

$\Delta p/\Delta c$ is the marginal rate of substitution of corn for potatoes. Similarly, .39 may be described as the marginal rate of substitution of corn for potatoes for the capital requirements.

Feasible and optimum plans

To start with, point b in Figure 1 can be selected as a feasible plan. Under that plan, all the area will be under potatoes and nothing under corn. Now we would like to know whether this is an optimum plan or whether there are more profitable plans. This can be ascertained from the following criterion equation:

$$\Delta Z_0 = R_c - R_p \cdot \frac{\Delta p}{\Delta c} \tag{5.6}$$

where Z_0 is the increase in profits associated with every unit increase in corn. R_c and R_p are the net revenues of corn and potatoes respectively and $\frac{\Delta p}{\Delta c}$ is the marginal rate of substitution of corn for potatoes. We know the slope of b c from Equation 5.4. With the net revenues of potatoes at \$ .834 per acre and at \$ .724 per acre of corn, we can rewrite Equation 5.6 as

$$\Delta Z_0 = .724 - (.834 \times .39) = 0.40 \tag{5.7}$$

Since Z_0 is positive, profit can be increased by substituting corn for potatoes. Again applying the criterion equation, we test whether the move from c to a is profitable or not. The increase in profits may be computed as

$$\Delta Z_0 = .724 - (.834 \times 1) = -0.11 \tag{5.8}$$

The negative profits imply that it is not profitable to move from the point c. The point c with 5.59 acres under potatoes and 54.41 acres under corn is not only a feasible plan but also an optimum plan.

The optimum program, it was seen, was at the corner c, the point of intersection of the isoresource curves. Points b and c are also corners and the solution or the optimum program revolves around finding that corner which yields maximum profits or intersects the highest isorevenue curve. The simplex procedure developed by Dantzig (1949) essentially uses the above principles. However, in the simplex table, disposal activities are included for nonuse of some of the activities.

Having considered the different approaches to economic analysis and the basic concepts of activity analysis and linear programming, we proceed to consider the generalized activity models and the linear programming formulation within input-output framework.

Generalized activity model

The Leontief system is based on the idea that the choice of technology, source of supply and pattern of demand are independent of the outcome of analysis. The system assumes that choices are not dependent on the level of output in each sector and therefore, can be fixed in advance. In this part, we cast the input-output model in the linear programming framework and thereby introduce the element of choice in the model. The programming approach considers alternative sources of supply as separate activities and the level at which each is utilized becomes a variable in the model. The system, therefore, is not unique and has many possible solutions. Another advantage is the criterion for preferring one solution to another, which may be maximization of welfare, national income or output levels or cost minimization. As discussed earlier, the maximizing behavior, which is central to economic theory, is completely ignored in the Leontief system. By formulating a programming model, we introduce choice and a certain criterion for preferring one solution to another and hence, indirectly incorporate the economizing assumption in the Leontief model. Though a planning authority is interested in knowing the network of interdependencies in the economy, he is more interested in finding out the output levels or final demands under different changing situations. The programming formulation aids the planner in arriving at the optimal choice within the resource limitation, commodity levels and structural constraints.

In the input-output model, we have a number of industries or sectors of production. As the model generally assumes fixed proportions of inputs to output, each of the industries can be considered as an activity. Any activity j may be represented as a column vector as shown in Equation 5.9, where A_j is the column vector of input-output coefficients a_{ij}'s for activity j. Positive coefficients in the vector denote outputs and negative coefficients describe inputs.

Assuming that there are n activities, the activities

$$A_j = \begin{bmatrix} a_{1j} \\ a_{2j} \\ a_{3j} \\ \vdots \\ a_{mj} \end{bmatrix} \qquad (5.9)$$

can be expressed in a matrix form as

$$A = (A_1, A_2, \ldots, A_n) = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix} \qquad (5.10)$$

where A is the matrix of technical coefficients of n activities that produce m commodities. In the Leontief input-output matrix, the number of activities and the number of commodities are equal, i.e., n = m, and therefore the model is deterministic with no choice of activity. In any planning situation, it will be almost impossible to anticipate a particular set of proportions and the outputs as detailed in a Leontief matrix. The planner would like to choose from the different alternatives the optimum that maximizes welfare or national income or those outputs that could be produced with minimum cost. So in the activity model or linear programming the output levels (gross or net) become the activity levels. Using the same notations used in input-output analysis, there are a set of activity levels (X_1, X_2, ..., X_n), with the input-output relation as

$$X_{ij} = a_{ij} X_j \tag{5.11}$$

The final demand (Y) constitutes the restrictions, which are taken as given from extraneous sources. There is one restriction for each of the m sectors, industries or commodities. The column vector of constraints may be represented as

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ Y_3 \\ \cdot \\ \cdot \\ \cdot \\ Y_m \end{bmatrix} \tag{5.12}$$

The linear programming problem consists essentially in the choice of the activities. At this stage, mention may be made about the assumption of linearity, which is basic to the theory of linear programming. It would be appropriate to quote Tinbergen in this context:

> First it is a well known mathematical proposition that almost any function may be approximated by linear functions over small intervals. The exceptional functions for which this proposition does not hold need not interest us here at all. Most economists would not be aware of their existence.
>
> The second reason why I think linear relations are not so ridiculous, is that observation simply teaches us that they occur.
>
> Apart from these reasons, is it not natural to begin any attempt at analyzing the economic mechanism by making the simplest assumption compatible with general theory?
>
> In addition to all this there is one theoretical reason why for great masses of individuals, the joint reaction may be much more than individual reaction will be. (1958)

Having considered the logic in favor of linear relationships, we can represent the linear programming problem as

$$\text{Maximize } C = \Sigma \; c_j \, X_j \tag{5.13}$$

subject to

$$\sum_j a_{ij} \, X_j \leq Y_i \tag{5.14}$$

$$X_j \geq 0 \tag{5.15}$$

where X_j is the activity level, a_{ij}'s are input-output coefficients. Y_i's are final demands or specified constraints. A linear programming model seeks to maximize or minimize some linear function of activities subject to certain restrictions. The choice of the objective function depends on the welfare function, the community wants to maximize. The method of reaching the optimum solution was discussed earlier in the numerical example.

System of linear equations and linear programming

In a system of linear equations, a unique solutions exists, if the number of linearly independent equations is equal to the number of unknowns. There is more than a solution, if the number of variables exceeds the number of equations. If the number of equations is m and there are n unknowns ($n > m$), the system has ($n - m$) fold infinity of solutions and the system is considered to be underidentified. The system is overidentified when the number of equations exceeds the variables or the degrees of freedom is negative. This implies that we have more restrictions than the variables, which is not consistent. In a crude way, one could ascertain whether a solution exists or not, by counting the number of variables and the number of equations. In the linear programming formulation, the number of variables exceeds the number of equations. The Leontief input-output system will yield a deterministic solution, since the variables just equal the number of equations. Therefore, in casting the Leontief model in the linear programming framework, additional variables have to be introduced for an optimum solution.

A Linear Programming Model within the Input-output Framework

One of the objectives of development programming is to obtain a wide perspective of the economic development of the country so as to establish coordinated production targets compatible with the stability of the system. A detailed perspective affords information on the different sectors and their capacities and also provides the necessary base for certain criteria relating to the establishment of financial, monetary foreign trade and other policies, that will lead to the success of the plans or objectives. A planner, for example, can formulate the production targets taking into account the necessary rate of development within the limitations of capital and other resources. In this study, an attempt has been made to develop a linear programming model within the input-output framework to deal with the problem of allocation of capital resources during the Third Five Year Plan period for the Indian economy.

For our model, the 1960-61 input-output projections (Appendix B) published by the Institute of Public Opinion (ca. 1960) were used. There are nineteen sectors in the table, but they had to be aggregated to ten sectors to meet the available computational facilities. As the sectors are more detailed, the input-output coefficients get smaller and the rounding errors tend to get magnified. In the aggregated table, food crops include wheat and other cereals and fibre products include wood, jute, textiles and leather and rubber. Non-ferrous metals include chemicals, besides non-metals. Mining was aggregated with ferrous metals and its products. Capital intensive sectors, as power and transport were combined to form a single sector.

Programming formulation

The model seeks to maximize the total gross national income at the end of the Third Plan period, subject to the structural restrictions and certain output levels targetted for the plan period. In addition, the total capital available during the Third Plan is limited and the estimated outlay of ten thousand crores of rupees forms the resource constraint. The problem is, therefore, to find the optimum output levels during the plan period with the dual objective of maximizing the national income or outputs and allocating the capital to the various activities in an optimal manner.

The mathematical version of the model may be represented as shown on the following page, where ΔX_1 is the difference between the targetted outputs in 1965-66 and the actual outputs attained in 1960-61 in the food crops sector. Similarly, ΔX_2, ΔX_3, ..., ΔX_{10} are the increases in outputs in industrial crops sector, livestock products and so on. The a_{ij}'s are the input-output coefficients for the year

$$\text{Max } F = \Delta X_1 + \Delta X_2 + \Delta X_3 + \Delta X_4 + \Delta X_5 + \Delta X_6 + \Delta X_7 + \Delta X_8 + \Delta X_9 + \Delta X_{10}$$

subject to

$$\begin{aligned}
a_{11}\Delta X_1 + a_{12}\Delta X_2 + \dots + a_{1,10}\Delta X_{10} &\leq \Delta X_1 \\
a_{21}\Delta X_1 + a_{22}\Delta X_2 + \dots + a_{2,10}\Delta X_{10} &\leq \Delta X_2 \\
\vdots \qquad\quad \vdots \qquad\qquad\quad \vdots \qquad &\quad \vdots \\
a_{10,1}\Delta X_1 + a_{10,2}\Delta X_2 + \dots + a_{10,10}\Delta X_{10} &\leq \Delta X_{10}
\end{aligned} \tag{5.16}$$

$$b_1\Delta X_1 + b_2\Delta X_3 + \dots + b_{10}\Delta X_{10} \leq K$$

and $\Delta X_{i-10} \leq \text{Max } \Delta X_{i-10}, \quad i = 11, 12, \dots, 20$

$$\Delta X_i \geq 0$$

1960-61 and we assume that these coefficients do not change over the planning period. The b_i's are the capital-output ratios and K is the total investment made during the Third Plan period. An equal weight of one has been given for all the ten activities so that our aim is to maximize the gross national output or the national income.

The model described in System 5.16 can be written

$$\text{Max } F = \Delta X_1 + \Delta X_2 + \Delta X_3 + \dots + \Delta X_{10}$$

subject to

$$
\begin{array}{llllll}
(1-a_{11})\Delta X_1 & - a_{12}\Delta X_2 & - a_{13}\Delta X_3 & - \dots - a_{1,10}\Delta X_{10} - Y_1 & = & 0 \\
-a_{21}\Delta X_1 & + (1-a_{22})\Delta X_2 & - a_{23}\Delta X_3 & - \dots - a_{2,10}\Delta X_{10} - Y_2 & = & 0 \\
-a_{31}\Delta X_1 & - a_{32}\Delta X_2 & + (1-a_{33})\Delta X_3 & - \dots - a_{3,10}\Delta X_{10} - Y_3 & = & 0 \\
\vdots & \vdots & \vdots & \vdots & & \vdots \\
-a_{10,1}\Delta X_1 & -a_{10,2}\Delta X_2 & - a_{10,3}\Delta X_3 & - \dots + (1-a_{10,10})\Delta X_{10} - Y_{10} & = & 0 \\
 & & & & & \\
-b_1\Delta X_1 & - b_2\Delta X_2 & -b_3\Delta X_3 & - \dots - b_{10}\Delta X_{10} - c & = & -K
\end{array}
\qquad (5.17)
$$

and

$$
\begin{array}{lcl}
\Delta X_1 + p_1 & = & \text{Max } \Delta X_1 \\
\Delta X_2 + p_2 & = & \text{Max } \Delta X_2 \\
\vdots \quad \vdots & & \vdots \\
\Delta X_{10} + p_{10} & = & \text{Max } \Delta X_{10}
\end{array}
$$

$$\Delta X_i \geq 0$$

including the slack variables, which would render the inequalities as equalities.

It would be seen from System 5.17 that the final demands Y_i's are obtained as slack variables. Similarly, c represents the slack for unused capital and Max ΔX_1, Max ΔX_2, ..., Max ΔX_{10} indicate the maximum output levels or upper bound constraints. The slacks p_1, p_2, ..., p_{10} denote the amount by which the upper bound levels are not met. Since the model is described in terms of increases (ΔX's), the lower bound is zero and is built within the model.

Description of the model

In India, the Planning Commission (1961) has estimated that the investment during the Third Five Year Plan will be to the tune of ten thousand crores of rupees. So it was taken that the capital available for new investment during the five years 1961-65 would be ten thousand crores. The output levels that have been attained at the end of Second Five Year Plan, i.e., in 1960-61, were taken as the minimum levels. For the most part, the minimum levels were adopted from the 1960-61 projections of the inter-industry table published by the Institute of Public Opinion (*ca*. 1960). From the estimates of investments, growth of capacities, demand for products consequent on population growth, increase in standards of living, et cetera, maximum outputs have been projected by the Planning Commission (1961). Instead of adopting these levels as such, we thought it better to calculate approximate increases during 1961 to 1966 in terms of percentages, since at the time of planning, much ahead of the period in which it is put into operation, it would be rather operational to anticipate increases as percentages rather than fixing any particular level of output. These percentages and the calculated output levels are given in Table 19. Since we are interested in allocating the capital resources for increases in output, the lower bound, as indicated, was taken as the production levels already attained in 1960-61. The increases in the activity levels (ΔX's) are, therefore, the differences between the anticipated outputs in 1965-66 and the outputs already attained at the end of the Second Five Year Plan, i.e., 1960-61.

Our aim is to maximize the gross national income or gross output. In the process of planning in a less developed economy, a balanced growth is needed and therefore, equal weights were given for all the sectors.

The technological changes in less developed economies take place rather very slowly due to many causes, one of them being institutional rigidities. So, in our model, we have assumed that the technology is not likely to change from the 1960-61 level and hence have adopted the input-output coefficients (a_{ij}'s) of the base period. The aggregated ten sector technology matrix and the input-output coefficients

Table 19. Estimated increases in outputs in the Third Plan (output in crores of rupees)

Sector	Output in 1960-61	Percentage increase anticipated	Output at the end of 1965-66	Estimated increase in output (ΔX)
	1	2	3	4
1. Food crops	4313.45	30	5607.46	1294.01
2. Industrial crops	1072.28	30	1393.96	321.68
3. Livestock products	1758.91	25	2110.69	351.78
4. Other agricultural products	3095.10	30	4023.63	928.53
5. Food and vegetable oils	1167.78	30	1518.11	350.33
6. Fibre products	1663.98	40	2329.57	665.49
7. Mining, metals and its products	1092.81	37	1497.87	405.03
8. Non-ferrous metals and chemicals	240.72	50	361.08	120.36
9. Power and transport	1682.74	50	2524.11	841.37
10. Construction	551.40	31	722.13	170.73

(a_{ij}'s) for the year 1960-61 are given in Tables 20 and 21 respectively.

Capital-output ratios

The amount of capital required to produce one unit of gross output was assumed to be the capital-output ratio for the purpose of our model. It was further assumed that the capacities are fully utilized in the base year 1960-61. In the absence of reliable estimates, the capital-output ratios, applied by Sandee (1960) for his Demonstration Planning model for India were approximated to suit the 10 sectors in the model. Certain estimates of the capital-output ratios for agriculture were also available with the Planning Commission. The ratio for agriculture was estimated by the Planning Commission by dividing the capital stock with the net domestic product. The capital output-ratio of the latest available year (1957) was calculated as follows:

$$\frac{\text{capital stock}}{\text{domestic product}} = \frac{6853}{5520} = 1.2414$$

The five year average ratio was quite close, though lower, but since it was felt that the condition in 1960 will be approximately closer to 1957 than earlier years, the 1957 figure, the latest available, was adopted. Similarly, for chemicals, textiles, jute and vegetable oils in the 19 sector table, the estimates provided by the Reserve Bank of India

Table 20. Transaction matrix for India, 1960-61 (in 10 millions of rupees)

	1[a]	2	3	4	5	6	7	8	9	10	Total inter-industry uses
1[a]	214.59	2.02	25.91	3.46	26.55	2.31	-	-	88.37	-	363.21
2	3.02	63.81	-	-	499.46	363.75	-	-	-	-	930.04
3	88.20	12.70	97.81	55.70	64.69	64.89	-	-	124.43	-	508.42
4	176.19	-	537.49	216.48	52.90	-	-	0.55	118.83	-	1102.44
5	8.04	-	24.45	3.17	37.15	2.93	-	16.39	6.19	-	98.32
6	36.74	8.77	-	21.94	8.82	167.29	1.91	15.67	40.98	42.87	344.99
7	0.20	-	-	5.77	12.90	21.74	217.75	29.28	94.06	56.73	438.43
8	2.05	-	-	5.77	7.59	24.04	9.79	14.92	1.25	92.62	158.03
9	54.02	43.25	16.03	69.27	44.48	78.51	102.56	53.09	106.63	30.71	598.55
10	0.20	-	-	-	6.14	4.50	1.32	7.65	7.60	-	27.41

[a]Sectors 1 to 10 are the same as described in Table 19, page 136.

Table 20 (Continued)

	Exports	Government expenditure and investment	Consumption	Total final demand	Total gross product
1[a]	-	-	3950.24	3950.24	4313.45
2	31.00	-	111.24	142.24	1072.28
3	14.55	-	1285.94	1249.94	1758.91
4	240.00	1.00	1751.66	1991.00	3095.10
5	55.00	-	1014.46	1069.46	1167.78
6	315.75	32.80	970.44	1318.99	1663.98
7	92.50	341.35	220.53	754.38	1092.81
8	10.00	2.00	71.44	83.44	240.72
9	14.00	30.00	1040.19	1084.19	1682.74
10	-	524.00	-	524.00	551.40

Table 21. Input-output coefficients

Sector	1	2	3	4	5
1. Food crops	0.0497	0.0018	0.0147	0.0011	0.0227
2. Industrial crops	0.0007	0.0595	0.0000	0.0000	0.4276
3. Livestock products	0.0204	0.0118	0.0556	0.0179	0.0553
4. Other agricultural products	0.0408	0.0000	0.3055	0.0699	0.0452
5. Food and vegetable oils	0.0018	0.0000	0.0138	0.0010	0.0318
6. Fibre products	0.0085	0.0081	0.0000	0.0070	0.0075
7. Mining, metals and its products	0.0000	0.0000	0.0000	0.0018	0.0110
8. Non-ferrous metals and chemicals	0.0004	0.0000	0.0000	0.0018	0.0064
9. Power and transport	0.0125	0.0403	0.0091	0.0223	0.0380
10. Construction	0.0000	0.0000	0.0000	0.0000	0.0052

Table 21 (Continued)

Sector	6	7	8	9	10
1. Food crops	0.0013	0.0000	0.0000	0.0525	0.0000
2. Industrial crops	0.2185	0.0000	0.0000	0.0000	0.0000
3. Livestock products	0.0389	0.0000	0.0000	0.0739	0.0000
4. Other agricultural products	0.0000	0.0000	0.0022	0.0706	0.0000
5. Food and vegetable oils	0.0017	0.0000	0.0680	0.0360	0.0000
6. Fibre products	0.1005	0.0000	0.0650	0.0243	0.0777
7. Mining, metals and its products	0.0130	0.1992	0.1216	0.0558	0.1028
8. Non-ferrous metals and chemicals	0.0144	0.0089	0.0619	0.0070	0.1679
9. Power and transport	0.0471	0.0938	0.2205	0.0633	0.0556
10. Construction	0.0027	0.0012	0.0317	0.0045	0.0000

were used. In our 10 sector model, the ratios are weighted averages of the 19 sector classification described in the following pages. The capital-output ratios for the ten sectors in our model are given in Table 22.

Table 22. Capital-output ratios

Sector	Capital-output ratio
1. Food crops	1.21
2. Industrial crops	1.21
3. Livestock products	1.21
4. Other agricultural products	1.21
5. Food and vegetable oils	0.59
6. Fibre products	1.17
7. Mining, metals and its products	2.36
8. Non-ferrous metals and chemicals	2.03
9. Power and transport	5.11
10. Construction	0.22

The simplex solution to the problem indicated that the upper bounds or targets shown in Column 4 of Table 19 could be attained except in the case of power and transport (Sector 9). In the case of power and transport, the optimum increase was found to be output valued at 831.07 crores of rupees, instead of the targetted level of 841.37 crores.

Having obtained the increases in outputs (ΔX's) from the simplex solution, we computed the output levels at the end of the Third Plan, *viz.*, 1955-56. The output levels (X's) are the same described in Column 3 of Table 19 except in the case of power and transport. The optimum output of power and transport from the solution is 2513.81 crores of rupees, instead of the targetted figure of 2524.11. From the optimum output levels, we computed the optimum levels of final demand and inter-industry demand at the end of the Third Five Year Plan. The transaction matrix of the optimums is presented in Table 23.

Though our estimates of outputs were very rough, they were generally higher than the estimates of the Planning Commission. Since the sectors are highly aggregated, only approximate percentages of the possible increases could be guessed. Despite the inherent deficiencies in the data, it appears that higher targets than those estimated by the Planning Commission seemed feasible. The optimum solution lends support to the criticism of Surendra J. Patel (1961) that the targets of the Planning Commission are conservative and higher levels of production can be attained with the intended level of investment of ten thousand crores of rupees during the Third Plan period.

Table 23. Projection of the transaction matrix for 1965-66 based on the optimum output levels (in 10 millions of rupees)

	1[a]	2	3	4	5	6	7	8	9	10
1[a]	278.69	2.51	31.03	4.43	34.46	3.43	-	-	131.97	-
2	3.92	82.94	-	-	649.14	509.41	-	-	-	-
3	114.39	16.45	117.35	71.97	83.95	90.62	-	-	185.77	-
4	228.78	-	644.82	281.25	68.62	-	-	0.79	177.47	-
5	10.09	-	29.13	4.02	48.28	3.96	-	24.55	9.05	-
6	47.66	11.29	-	28.16	11.39	234.12	2.55	23.47	61.09	56.11
7	-	-	-	7.24	16.69	30.28	298.37	43.91	140.27	74.23
8	2.24	-	-	7.24	9.71	33.55	13.33	22.35	1.76	121.25
9	70.09	56.18	19.21	89.73	57.69	109.72	140.49	79.62	159.12	40.15
10	-	-	-	-	7.89	6.29	1.79	11.45	11.31	-

[a]Sectors 1 to 10 are the same as described in Table 19, page 136.

Table 23 (Continued)

	Total (1-10)	Final demand	Gross output
1[a]	486.42	5121.04	5607.46
2	1245.41	148.55	1393.96
3	680.50	1430.19	2110.69
4	1401.73	2621.90	4023.63
5	129.08	1389.03	1518.11
6	475.84	1853.73	2329.57
7	610.99	886.85	1497.84
8	211.43	149.65	361.08
9	822.00	1691.81	2513.81
10	38.73	683.40	722.13

Limitations of the model

One of the weak links in the model is the assumption that the technology remains constant. Though it could be argued that changes in technology take root rather very slowly in less developed economies, the planning in the past has generated certain expansion of industries and the pumping in of further investment is bound to alter the input mix. The model does not take care of such possible changes in input coefficients. However, if definite information of changes in input coefficients of some strategic industries are available as a result of surveys, they could be incorporated in the model without difficulty.

A more important weakness in the model is that there is only one effective resource restriction, *viz*., the capital-output ratios. Additional restrictions could not be imposed on the model for want of adequate data. One of the possible ways to improve the practical utility of the model is to impose an additional restriction on the employment potential. Here also, the labor force may be divided into skilled labor and unskilled labor. In under-developed economies, there is no scarcity of unskilled labor in view of chronic unemployment and underemployment. What is dear is the skilled labor and if we could obtain capital-labor ratios of skilled personnel, it could form an ideal restriction and might enhance the practical applicability of the model for planning purposes.

Tinbergen (1958), Sandee (1960) and Chenery (1958) have explored the possibility of applying accounting prices or shadow prices rather than market prices. Tinbergen (1958) has postulated a hypothesis that market prices of a number of commodities, particularly those of factors of production, as capital, labor and foreign exchange, often differ from their 'intrinsic value' or accounting prices. The important reason for this divergence is attributed to the fundamental disequilibrium as widespread unemployment in less developed economies due to the absence of complementary means of production. In spite of large scale unemployment, the market wages are likely to be higher due to activities of the trade unions and certain welfare measures imposed by the governments. If we accept that the market rates do not reflect their intrinsic value, it might be desirable to give different weights to the maximizing function. While the calculation of optimum weights is, no doubt, difficult in view of the absence of data depicting the extent of disequilibrium, certain approximations could be tried. This approach, it is possible, might render some of the projects more profitable than at market prices. This is an important aspect for future workers to pursue.

Our primary concern has been to develop a linear programming model within the input-output framework. The data used and the results obtained are, therefore, of secondary importance. For any meaningful programming solution the input-output coefficients should be reliable and the resource constraints should be effective and binding. Capital is not the only scarce resource in an underdeveloped economy. Therefore, no claim is laid about the applicability of the model to actual planning situations. The model is just illustrative and has to be modified for any specific planning formulation by the introduction of additional resource and other constraints.

A Variable Capital Programming Model

It is common knowledge that capital resources for investment in all less developed economies are very scarce and most of the developing economies heavily depend on foreign aid for their economic development. As the development progresses, there is an aggregate increase in the capital inputs. For instance, in the United States there has been a phenomenal increase in the input of capital items for agriculture. Heady (1962) has estimated that machinery and equipment used by 1960 were three times that of 1940. As the technology advanced, inputs of capital items, as the new crop varieties, insecticides, antibiotics, pure-bred animals, et cetera, had also increased by four to five times in the last twenty years. So the problem that faces the planners of a country or an individual farmer is to find an optimum plan, under conditions of varying levels of capital. If a certain amount of capital is available, then a discrete optimum can be found assuming clearly defined resource and price situations. But, when the level of capital availability is an uncertainty and is linked to forces outside the control of the planner or the farmer, then a single optimum will not be meaningful. There is, therefore, the need for optimum plans for each of a series of different capital levels. In the variable resource programming, we seek a series of solutions assuming a different initial level of capital.

Logic of variable resource (capital) programming

The essential feature of the variable capital programming is that capital is always invested in the activity with the highest marginal product. Or, in other words, every rupee invested yields the highest possible income and for any level of capital availability, the amount of income is a maximum. As the income is a maximum for each level of capital, the program is an optimum for each of the levels. This type of variable capital programming is also known as continuous programming since the assumptions, as Heady (1958) has pointed out, prevent calculations with increasing marginal products. If, for example, there is a fixed plant, this investment has zero marginal product and will not be undertaken until all other investment opportunities have been exhausted.

Modified simplex procedure

Heady (1958), Candler (1956), Boles (1956) and others have suggested a modified simplex method for computing optimum solutions for each of the varying levels of capital. The modified method allows a planner or a farmer to follow all the changes in the optimum plan, as the availability of capital increases from zero to any higher level. As discussed earlier, the principle again is to invest each dollar or rupee where its marginal product is highest. If in the simplex table we could identify the activity with the highest marginal product, then our problem is solved, since that row can be selected as the outgoing row. For identifying the activity with the highest marginal productivity of capital, we include a new row, which Heady (1958) designates as the 'decision row' in the simplex table. The $\bar{D}$ (decision row) is obtained by dividing the negative Z-C row (profit row) by the appropriate input-output coefficients for capital. The highest negative element in the new criterion $\bar{D}$ row is selected as the activity. For purposes of illustration, we will take the example provided by Heady (1958). Instead of assuming that capital is available at a single level, we will make the additional assumption that capital is available at varying levels and seek optimal solutions for each of the levels of capital availability. An illustrative modified simplex procedure for variable capital is provided in Figure 2.

In the simplex table, we can note some of the important differences between the ordinary simplex procedure and the modified simplex method for variable capital (resource) programming. The resource level, which is considered to vary is recorded at the zero level. In the present case, capital is the variable resource, while all the other resources have fixed levels. An important departure from the normal simplex method is the calculation of the decision row. The element in the column P_4, for example, is obtained by dividing the element in the Z-C row, viz., 220, by the input-output coefficient of capital for that activity, i.e., 1.1. The highest element in the decision row ($\bar{D}$) denotes the marginal productivity of capital for that activity is largest. In the example P_5 is the row with highest marginal productivity. Another deviation from the simplex procedure is in the computation of R ratio for limiting resources. While in the normal simplex method, the smallest R is taken as the outgoing row, in the modified simplex procedure, the outgoing row is given by the smallest R ratio "other than capital". The capital is never selected as the outgoing row. The result of this will end up in negative levels for capital in successive iterations. Since capital is not a fixed resource, the equations for the production possibilities will not include the activities' requirements for this resource,

			C →		0	0	-0	0	0	-50	-70	70	220	130	-5	Σ	
C_b				Resource or activity level B	Disposal activities					Real activities						or P_{check}	R
					Capital P_7	Land P_8	Hog housing P_9	Hay P_{10}	Corn P_{11}	Rotation 1 P_1	Rotation 2 P_2	Cattle P_3	Hogs P_4	Corn selling P_5	Hay buying P_6		
0		Capital	P_7	0	1	0	0	0	0	.6	.6	1	1.1	0	.05	4.35	--
0		Land	P_8	120	0	1	0	0	0	3	4	0	0	0	0	128	--
0		Hog housing	P_9	15	0	0	1	0	0	0	0	0	1	0	0	17	--
0		Hay	P_{10}	0	0	0	0	1	0	-2	-2.5	.8	1.2	0	-1	-2.5	--
0		Corn ←	P_{11}	0	0	0	0	0	1	-1.4	-1.5	.5	1.1	1	0	.7	0
			Z-C	0	0	0	0	0	0	50	70	-70	-220	$\boxed{-130}$	5	-295	
			1-Σ	-134	0	0	0	0	0	-49.2	-69.6	68.7	216.6	130	-3.05	148.45	
			$\bar{D}$		•	•	•	•	•	•	•	-70	-200	- —	•		
0			P_7	0	1	0	0	0	0	.6	.6	1	1.1	0	.05	4.35	0
0		←	P_8	120	0	1	0	0	0	3	4	0	0	0	0	128	40
0			P_9	15	0	0	1	0	0	0	0	0	1	0	0	17	--
0			P_{10}	0	0	0	0	1	0	-2	-2.5	.8	1.2	0	-1	-2.5	--
130		→	P_5	0	0	0	0	0	1	-1.4	-1.5	.5	1.1	1	0	.7	--
			Z-C	0	0	0	0	0	130	$\boxed{-132}$	-125	-5	-77	0	5	-204	
			1-Σ	-134						132.8		3.7	73.6		-3.05		
			$\bar{D}$		•	•	•	•	•	-220	-208.333	-5	-70	•	•		
0			P_7	-24	1	-.2	0	0	0	0	-.2	1	1.1	0	.05	-21.25	--
-50		→	P_1	40	0	.333	0	0	0	1	1.333	0	0	0	0	42.667	--
0		←	P_9	15	0	0	1	0	0	0	0	0	1	0	0	17	15
0			P_{10}	80	0	.666	0	1	0	0	.166	.8	1.2	0	-1	82.834	67
130			P_5	56	0	.466	0	0	1	0	.366	.5	1.1	1	0	60.434	51
			Z-C	5280	0	43.956	0	0	130	0	50.956	-5	$\boxed{-77}$	0	5	5428.044	
			1-Σ	-5446								3.7	73.6		-3.05		
	B'		$\bar{D}$		•	•	•	•	•	•	•	-5	-70	•	•		
0	59.5		P_7	-40.5	1	-.2	-1.1	0	0	0	-.2	1	0	0	.05	-39.95	--
-50	40		P_1	40	0							0			0		--
220	15	→	P_4	15	0	0	1	0	0	0	0	0	1	0	0	17	--
0	62	←	P_{10}	62	0	.666	-1.2	1	0	0	.166	.8	0	0	-1	62.434	77.5
130	39.5		P_5	39.5	0	.466	-1.1	0	1	0	.366	.5	0	1	0	41.734	79
	6435		Z-C	6435	0	43.956	77	0	130	0	50.956	$\boxed{-5}$	0	0	5	6737.044	
	-6650		1-Σ	-6550								3.7			-3.05		
			$\bar{D}$		•	•	•	•	•	•	•	-5	•	•	•		
0			P_7	-118	1	-1.033	.4	-1.25	0	0	-.408	0	0	0	1.3	-117.992	--
-50			P_1	40											0		--
220			P_4	15											0		--
70		→	P_3	77.5	0	.833	-1.5	1.25	0	0	.208	1	0	0	-1.25	78.042	--
130		←	P_5	.75	0	.05	-.35	-.625	1	0	.262	0	0	1	.625	2.713	1.2
			Z-C	6622.5	0	48.121	69.5	6.25	130	0	51.996	0	0	0	$\boxed{-1.25}$	7127.254	--
			1-Σ	-6836.75											1.575		
			$\bar{D}$		•	•	•	•	•	•	•	•	•	•	-.962		
0			P_7	-119.56													
-50			P_1	40													
220			P_4	15													
70			P_3	79													
-5		→	P_6	1.2	0	.08	-.56	-1	1.6	0	.419	0	0	1.6	1	4.341	
			Z-C	6824	0	48.221	68.8	5	132	0	53.52	0	0	2	0	7132.68	
			1-Σ	-6838.64													
70		→	P_3	59.5	1	-.2	-1.1	0	0	0	-.2	1	0	0	.05	60.05	
-50			P_1	40	0												
220			P_4	15	0												
0			P_{10}	14.4	-.8												
130			P_5	9.75	-.5												
			Z-C	6732.5	5	42.956	71.5	0	130	0	49.956	0	0	0	5.25	7037.294	
			1-Σ	-6870.15													

Figure 2. An illustration of variable capital programming

but only the requirements of fixed resources and hence the negative levels for capital are consistent with any feasible plan. The negative level of capital simply means that if the capital had been at such and such a level, the plan in that iteration is feasible. For example, in Section 3 of the table (Figure 2), capital is -24, which denotes that the plan is optimum for the capital availability of $2400.

One advantage with a variable resource programming formulation is the computation of optimum plans of a number of discrete resource supplies. In the simplex table provided in Figure 2, the third section describes the optimum plan for $4050 of capital (since one unit is taken in the example as $100 of capital). Let us now, say, be interested in a plan for $1000 of capital availability. Section 5 in the table will not be feasible since it requires $11,800 of capital. Now the capital is fixed and so we may select the capital row if it has the smallest R ratio. B' column in Section 4 shows the level of capital at disposal, i.e. (100 - 40.5 = 59.5). Excepting for this, the plan of other activities remains the same. The capital row has the lowest R and is picked as the outgoing row. The optimum plan for 10,000 dollars of capital is shown in the last section of the simplex table (Figure 2).

Having discussed the technique of variable capital programming, let us consider the application of the variable capital programming for a programming formulation within the input-output framework.

Description of the model

For the variable capital programming model, we have used the nineteen sector table for India for 1960-61 published by the Institute of Public Opinion (Appendix B). As described earlier in the structural analysis, the first five sectors in the table are primarily agricultural sectors. The rest of the sectors may be broadly considered as industrial sectors.

In any programming formulation, the utility of a model would entirely depend on its practicability and application to current pressing problems. In a country like India, with a population of four hundred and thirty-eight millions of people (1961), any planning authority would have to give utmost priority to agriculture for two main reasons. First of all, the country should strive to produce the minimum output needed to feed its population at the rate of at least sixteen ounces per day. Otherwise, food has to be imported from foreign countries, which would result in a considerable drain of the foreign exchange resources. Secondly, if agricultural outputs are not met, then meeting of the targets in other sectors may well become impossible due to the inefficiency of labor as a result of discontentment and undernourishment. So, in the present model, we maximized the profit function giving a weight of 10 for the first five agricultural sectors and a weight of 0.1 for the rest fourteen industrial sectors.

Lower constraints

As for the minimum levels of outputs that have to be met, it was considered that the levels of outputs at the 1960-61 would not be meaningful since those targets have been more or less attained nearly two years ago. In as much as we are in the second year of the Third Five Year Plan, it is to be expected that in several sectors, the outputs would now be higher than the 1960-61 level. In order that the model may be realistic, the lower bound for output levels were computed as fifty percent of the difference between the 1960-61 levels and the 1965-66 levels. For example, the output level for the wheat and other cereals during 1960-61 was 2214.84 crores of rupees. The estimated output for 1965-66 is 2879.29 crores. The increase estimated during Third Five Year Plan in the output of wheat and other cereals is 664.45 crores. Of this output, the minimum level was put at 50 percent of the anticipated increase plus the 1960-61 output level. Thus, the lower bound for wheat and other cereals was calculated as 2214.84 + 332.22 = 2547.06 crores of rupees. Similarly, the lower bound was calculated for all the other sectors and is presented in Column 3 of Table 24. The implication of fixing the minimum output levels at a level higher than the 1960-61 level is, that even if the worst comes, that is, even if an activity is not profitable, at least some increase would have been met and will not be merely at the 60-61 level. In the less developed economies, the interdependence between the industrial sectors and agricultural sectors is so low, that one cannot expect the industrial sectors to go up beyond the proportion needed for agricultural outputs, especially when we maximize the agricultural sectors with a higher weight than the industrial sectors.

Upper constraints

In the case of the maximum outputs that have to be attained at the end of the Third Five Year Plan, the output levels were calculated taking into account the production capacities that are available, the demand for products considering the population growth, et cetera. In general, the projected output levels of the planning commission provided a valuable guide in arriving at the maximum levels of output or the upper bounds desired to be attained. To give an idea of the manner in which output levels were arrived at by the Planning Commission, the output levels at the end of the Third Five Year Plan was estimated to increase by thirty percent over the 1960-61 levels in the case of the agricultural sectors except for livestock products. For livestock products, the increase was estimated at twenty percent. In the case of food manufactures, vegetable oils, construction and jute also, the estimated increase was around thirty percent. The increase

Table 24. Capital-output ratios and upper and lower constraints (outputs in crores of rupees)

Sectors 1	Capital-output ratio 2	Minimum levels 3	Maximum output levels in 1965-66 4
1. Wheat and other cereals	1.21	2547.06	2879.29
2. Rice	1.21	2413.40	2728.19
3. Industrial crops	1.21	1233.12	1393.96
4. Livestock products	1.21	1934.80	2110.69
5. Other agricultural products	1.21	3559.36	4023.63
6. Food manufactures	0.70	751.50	849.52
7. Vegetable oils	0.45	591.44	668.59
8. Wood	1.90	269.98	333.08
9. Jute	0.83	239.39	270.62
10. Textiles	1.01	1205.78	1325.27
11. Leather and leather goods	1.90	221.19	279.74
12. Fuel and power	6.40	1050.28	1104.12
13. Mining	2.40	215.82	265.62
14. Basic metal industries	2.35	320.99	451.48
15. Metal products	2.35	1240.15	1745.01
16. Non-metal products	1.90	133.99	170.46
17. Chemicals	2.12	274.38	405.86
18. Construction	0.22	636.76	722.13
19. Transport	3.25	934.13	1241.98

in sectors, like wood and mining was nearly sixty percent over the Second Plan targets. The estimated increase in the leather goods sector and non-metal products was 72 percent. The increases in both the basic metal industries sector and the metal products sector were taken at one hundred and thirty seven percent, as no separate breakdown of these independent sectors was available. This high increase in these two sectors is to be expected, since the four big steel plants are to come into full production during the Third Plan. Similarly, the increase in chemicals was also estimated at a high level of one hundred and eighty four percent, since the fertilizer production and other chemicals is expected to reach a new high. The targets or the upper bound constraints that were estimated are presented in Column 4 of Table 24. As contrasted from the earlier model, the output levels were calculated in a more detailed manner, checking at every stage with the Planning Commission's breakdown of estimates of corresponding sectors.

Capital-output ratios

Having computed the maximum levels, *viz.*, the upper bounds to be attained, as also the minimum constraints, the problem was one of maximizing total output subject to the structural constraints and output levels. An additional resource constraint was also introduced, since capital is one of the chief limiting factors in the development of the less developed areas. The capital-output ratios were computed for all the nineteen sectors. The interpretation of capital-output ratio may be considered as the amount of capital required for every unit of output. As the details of industries that were aggregated in each of the nineteen sectors were not precisely known, and as the capital-output ratios were not available for a number of sectors, they were approximated in several cases. In a programming formulation, Sandee (1960) had adopted certain capital-output ratios computed by Mukherjee and Shastry (1959) and these were adopted for most of the industrial sectors. The Reserve Bank of India had also estimated the capital-output ratios for vegetable oils, wood, jute and chemicals and these figures were used with respect to similar sectors in our table. In the case of agriculture, capital-output ratios were estimated by dividing the total capital stock estimated for agriculture with the domestic product for agriculture. This was the nearest approximation that could be made with the figures available at our disposal. The capital-output ratios for the nineteen sectors are given in Column 2 of Table 24.

It may be appropriate here to discuss briefly the concept of capital-output ratios. The amount of fixed capital needed to produce a unit of a commodity is generally known as the capital-output ratio of the production in question. Capital-output ratios are expressed in gross terms or in net terms. It is also expressed as value added or as contribution to national income, in which case, it expresses certain degree of netness. For our model, we have assumed that the capital-output ratios are based on gross output. A further assumption is that all the productive capacity is fully utilized and the new output, between 1960-61 and 1965-66, will have to come from the new capacity. It is also assumed that the normal level of inventories held is proportional to the output level and that a constant stock-output ratio is maintained. Sandee (1960) has discussed the inherent deficiencies of the capital-output ratios and has stated that the valuation of assets is a difficult problem. Again, the concept of full utilization is a vague concept and could lend itself to different interpretations.

Total investment in the Plan period

In our model, the total capital needed to produce the minimum levels of output (Column 3 of Table 24) amounted to 26933.49 crores of rupees. The outlay for investment during the Third Plan is estimated at 10,000 crores of rupees. Out of this total outlay, more than fifty percent is to come by way of foreign loans and aid. If for certain reasons outside the control of the planners this anticipated outlay does not become available, then arises the problem of pruning of the targets and setting up of priorities. In such a setting the planner is interested in finding the optimal solutions for different levels of capital availability. The variable capital programming provides optimal answers for such a predicament.

Mathematical formulation of the linear program

The mathematical formulation of linear programming may be expressed as

$$\text{Maximize } F = 10 \sum_{j=1}^{5} X_j + 0.1 \sum_{j=6}^{19} X_j$$

subject to

$$\begin{array}{llllll}
a_{11}X_1 & + a_{12}X_2 & + \cdots + & a_{1,19}X_{19} & \leq & X_1 \\
a_{21}X_1 & + a_{22}X_2 & + \cdots + & a_{2,19}X_{19} & \leq & X_2 \\
\vdots & \vdots & & \vdots & & \vdots \\
a_{19,1}X_1 & + a_{19,2}X_2 & + \cdots + & a_{19,19}X_{19} & \leq & X_{19} \\
b_1X_1 & + b_2X_2 & + \cdots + & b_{19}X_{19} & \leq & K
\end{array} \qquad (5.18)$$

$$X_{i-19} \leq \text{Max } X_{i-19} \quad (i = 20, 21, \ldots, 38)$$

$$X_{i-38} \geq \text{Min } X_{i-38} \quad (i = 39, 40, \ldots, 57)$$

$$X_i \geq 0$$

where X is the output level at the end of the Third Plan period, a_{ij}'s are input-output coefficients for the year

1960-61 and are shown in Table 25, and it is assumed that the technology remains constant over the planning horizon. The b_i's are capital-output ratios indicated in Column 2 of Table 24 and K is the total capital available for investment between 1960-61 and 1965-66. In this model, a weight of 100 for the agriculture sectors and a weight of 1 to the industrial sectors has been given. Assuming that the planners are interested in maximizing agricultural sectors, which are crucial to the development of the other sectors in a predominantly agricultural economy, higher weights were given for the agricultural sectors. Max X_{i-19} ($i = 20, 21, \ldots, 38$) are the upper constraints of targetted output levels indicated in Column 4 of Table 24 and Min X_{i-38} ($i = 39, 40, \ldots, 57$) are the lower bounds or minimum constraints given in Column 3 of Table 24.

The system described in Equation 5.18 can be written including the slack variables, which would render the inequalities as equalities, as in Equation 5.19, where Y_i is the final demand, which is obtained as a slack and the slacks p_i's show the disposals for maximum and minimum constraints. c is the slack variable for capital.

Table 26 describes the optimum outputs (Column 2) and final demand (Column 3). Column 4 indicates the amount by which the upper constraint is not met. Column 5 denotes the amount by which output has increased over the lower

Table 25. Input-output coefficients

Sectors[a]	1	2	3	4	5
1	.056428	-	-	-	-
2	.002546	.040012	.001884	.014731	.001179
3	-	.001439	.059509	-	-
4	.019911	.021014	.011843	.055608	.017996
5	.052275	.028786	-	.305581	.069943
6	-	-	-	-	-
7	.002357	.001344	-	.013901	.001024
8	.001038	.001053	-	-	.004850
9	.007644	.007290	.008179	-	.002239
10	-	-	-	-	-
11	-	-	-	-	-
12	.000190	.000386	-	-	.002239
13	-	-	-	-	-
14	-	-	-	-	-
15	-	.000095	-	-	.001864
16	-	-	-	-	-
17	.000470	.000481	-	-	.001864
18	-	.000095	-	-	-
19	.015288	.009020	.040335	.009114	.020141

[a]Sectors 1 to 19 are the same as described in Table 24, p. 157.

Table 25 (Continued)

Sectors[a]	6	7	8	9	10
1	.040629	-	-	-	.002126
2	-	-	-	-	-
3	.293673	.597997	-	.574242	.224811
4	.098993	-	.001305	-	.020934
5	.029044	.065954	-	-	-
6	.056849	-	-	-	.001151
7	-	-	-	-	.000359
8	.000842	.000292	.100783	.004660	.006030
9	.010375	.002605	-	.002594	.003019
10	-	-	-	.001777	.112585
11	-	-	.002320	.001201	.001068
12	.019327	.005794	.010151	.015948	.013394
13	-	-	-	-	-
14	-	-	-	-	-
15	.006687	.016586	.009377	.022914	.013836
16	.005662	-	-	-	-
17	.004015	.002294	.033788	.010520	.009408
18	.003581	.007389	.001208	.010232	.001951
19	.032151	.015283	.092179	.035067	.013578

Table 25 (Continued)

Sectors[a]	11	12	13	14	15
1	-	-	-	-	-
2	-	.082924	-	-	-
3	-	-	-	-	-
4	.257501	.124873	-	-	-
5	-	-	-	-	-
6	-	.000783	-	-	-
7	.007932	-	-	-	-
8	.032403	.026564	-	.000787	.001969
9	-	-	-	.001260	.000095
10	-	-	-	-	-
11	.032403	-	-	-	-
12	.085957	.011180	.018311	.114593	.005623
13	-	-	.016685	.143097	-
14	-	.000492	-	.001365	.186692
15	-	.013819	.014336	.028136	.057396
16	-	-	-	-	.002621
17	.028345	-	-	.005774	.009181
18	-	.003723	-	.006929	-
19	.021151	.063616	.171967	.135590	.026036

Table 25 (Continued)

Sectors[a]	16	17	18	19
1	-	-	-	-
2	-	-	-	.008364
3	-	-	-	-
4	-	-	-	-
5	.005582	-	-	.173148
6	-	.003219	-	-
7	-	.111469	-	.007883
8	.002436	.049612	.077747	.019219
9	.081599	.002099	-	-
10	-	-	-	.000379
11	-	-	-	.001544
12	.130620	.098943	.055694	.041775
13	.085659	.017004	.017864	-
14	-	.008257	.028437	.009325
15	.113671	.042194	.056583	.106952
16	-	.008117	.152104	-
17	.031462	.074592	.015869	.001821
18	.068304	.006438	-	.005668
19	.194052	.048702	-	.004998

$$\text{Max } F = 10X_1 + 10X_2 + \dots + 10X_5 + 0.1X_6 + 0.1X_7 + \dots + 0.1X_{19}$$

subject to

$$
\begin{array}{llllllll}
(1-a_{11})X_1 & - a_{12}X_2 & - a_{13}X_3 & - \dots - a_{1,19}X_{19} & & - Y_1 & = & 0 \\
-a_{21}X_1 & + (1-a_{22})X_2 & - a_{23}X_3 & - \dots - a_{2,19}X_{19} & & - Y_2 & = & 0 \\
-a_{31}X_1 & - a_{32}X_2 & + (1-a_{33})X_3 & - \dots - a_{3,19}X_{19} & & - Y_3 & = & 0 \\
\vdots & \vdots & \vdots & & \vdots & \vdots & & \\
-a_{19,1}X_1 & - a_{19,2}X_2 & - a_{19,3}X_3 & - \dots + (1-a_{19,19})X_{19} & & - Y_{19} & = & 0 \\
-b_1X_1 & - b_2X_2 & - b_3X_3 & - \dots - b_{10}X_{10} & & - c & = & -K
\end{array}
\qquad (5.19)
$$

$$X_{i-19} + p_{i-19} = \text{Max } X_{i-19} \quad (i = 20, 21, \dots, 38)$$

$$X_{i-38} - p_{i-38} = \text{Min } X_{i-38} \quad (i = 39, 40, \dots, 57)$$

$$X_i \geq 0$$

Table 26. Optimal solution for investment at 6000 crores or capital stock at 32933.49 crores of rupees

1	Optimum output levels 2	Final demand 3	Upper bound not met 4	Increase in output over and above minimum output levels 5
1. Wheat and other cereals	2812.10	2620.32	67.19	265.04
2. Rice	2413.40	2176.91	314.79	-
3. Industrial crops	1233.12	173.35	160.84	-
4. Livestock products	2110.69	1519.86	-	175.89
5. Other agriculture	3559.36	2220.43	464.27	-
6. Food manufacture	751.50	705.68	98.02	-
7. Vegetable oil	591.44	508.21	77.15	-
8. Wood	269.98	91.11	63.10	-
9. Jute	239.39	156.61	31.23	-
10. Textiles	1205.78	1069.24	119.49	-
11. Leather	221.19	210.33	58.55	-
12. Fuel and power	1050.28	801.33	53.84	-
13. Mines	215.82	138.77	49.81	-
14. Basic metal	320.99	59.05	130.49	-
15. Metal products	1240.65	930.45	504.36	-
16. Non-metals	133.99	27.40	36.47	-
17. Chemicals	274.38	181.85	131.48	-
18. Construction	636.76	601.82	85.37	-
19. Transport	964.13	447.24	277.85	-

constraints. It would be seen that outputs of wheat and other cereals has increased by 265.04 crores of rupees and livestock products by 175.89 crores of rupees. Even in the case of wheat, the output is lower by 67.19 crores of rupees than the maximum level. In all the other 17 sectors, outputs are at their minimum. The situation implies that the capital of six thousand crores of rupees is just sufficient to meet the increases in livestock and wheat and other cereals. With an investment of 6000 crores of rupees, only a little over fifty percent of the increase of output levels can be reached, since our minimum constraints were calculated as fifty percent of the difference between 1960-61 output levels and 1965-66 output levels plus the output levels attained in 1960-61. Now, we proceed to find out the output levels that could be met at the next higher level of capital availability. The optimum levels obtained after thirty-eight iterations at an investment level of 7218.58 crores of rupees or the total capital stock of 34152.07 crores of rupees are presented in Table 27.

From Table 27 it would be seen that the maximum output levels of all the five agricultural sectors are met with the investment level at 7218.52 crores of rupees. This plan indicates the capital investment required to meet the maximum targets for the five agricultural sectors and the minimums in the other fourteen industrial sectors, viz.,

Table 27. Optimal solution for investment at 7218.58 crores or total capital stock at 34152.07

1	Optimum output levels 2	Final demand 3	Upper bound not met 4	Increase in output over and above the minimums 5
1. Wheat and other cereals	2879.29	2683.72	-	332.23
2. Rice	2728.19	2478.09	-	314.79
3. Industrial crops	1393.96	324.16	-	160.84
4. Livestock products	2110.69	1501.64	-	175.89
5. Other agricultural crops	4023.63	2639.66	-	464.27
6. Food manufactures	751.50	705.68	98.02	-
7. Vegetable oils	591.44	507.15	77.15	-
8. Wood	269.98	88.47	63.10	-
9. Jute	239.39	151.45	31.23	-
10. Textiles	1205.78	1069.24	119.49	-
11. Leather	221.19	210.33	58.55	-
12. Fuel and power	1050.28	800.15	53.84	-
13. Mines	215.82	138.77	49.81	-
14. Basic metals	320.99	59.05	130.49	-
15. Metal products	1240.65	929.55	504.36	-
16. Non-metals	133.99	27.40	36.47	-
17. Chemicals	274.38	180.80	131.48	-
18. Construction	636.76	601.79	85.37	-
19. Transport	964.13	427.54	277.85	-

fifty percent of the estimated increase in the Third Five Year Plan period. In other words, the fourteen industrial sectors enter the solution at their minimum output levels and the five agricultural sectors at their maximum output levels.

The next plan considered is at an investment level of 8775.12 crores of rupees. This feasible plan was obtained after forty-eight iterations. The optimal output levels and final demands are presented in Table 28. At this level of investment, excepting the five sectors, viz., fuel and power, mines, basic metals, metal products and transport, all the other sectors enter the solution at their maximum levels. In the case of the five sectors, the outputs are at their minimum. One interesting aspect that may be noted from Table 24 is that the capital-output ratios of these five sectors, which enter at their minimums, are all fairly high. Therefore, if they are to be brought at their maximums higher weights may have to be assigned these sectors depending on their strategic importance in the course of the development of an economy.

We had so far considered the optimum levels of output at three intermediate levels of investment. If the entire outlay of 10,000 crores was available during the Third Plan period, then what will be the levels of output that would be met with the available capital investment?

Table 28. Optimal solution for investment at 8775.12 crores or total capital stock at 35708.61 crores of rupees

Sector	Optimum output levels	Final demand	Upper bound not met	Increase in output over and above the minimums
1	2	3	4	5
1. Wheat and other cereals	2879.29	2679.48	-	332.23
2. Rice	2728.19	2478.09	-	314.79
3. Industrial crops	1393.96	204.45	-	160.84
4. Livestock products	2110.69	1474.28	-	175.89
5. Other agri-cultural crops	4023.63	2631.52	-	464.27
6. Food manufactures	849.52	797.57	-	98.02
7. Vegetable oils	668.59	569.14	-	77.15
8. Wood	333.08	128.50	-	63.10
9. Jute	270.62	177.74	-	31.23
10. Textiles	1325.27	1175.22	-	119.49
11. Leather	279.74	266.67	-	58.55
12. Fuel and power	1050.28	765.84	53.84	-
13. Mines	215.82	131.88	49.81	-
14. Basic metals	320.99	-	130.49	-
15. Metal products	1538.14	1190.55	206.87	297.49
16. Non-metals	170.46	48.48	-	36.47
17. Chemicals	405.86	291.43	-	131.48
18. Construction	722.13	682.27	-	85.37
19. Transport	964.13	392.20	277.85	-

Table 29. Optimal solution for investment at 10,000 crores or total capital stock at 36933.49 crores of rupees

1	Optimum output levels 2	Final demand 3	Upper bound not met 4	Increase in output over and above the minimums 5
1. Wheat	2879.29	2679.48	-	332.23
2. Rice	2728.19	2477.28	-	314.79
3. Industrial crops	1393.96	204.45	-	160.84
4. Livestock products	2110.69	1474.28	-	175.89
5. Other agricultural crops	4023.63	2614.87	-	464.27
6. Food manufactures	849.52	797.57	-	98.02
7. Vegetable oils	668.59	568.38	-	77.15
8. Wood	333.08	126.14	-	63.10
9. Jute	270.62	177.56	-	31.23
10. Textiles	1325.27	1175.18	-	119.49
11. Leather	279.74	266.52	-	58.55
12. Fuel and power	1050.28	744.80	53.84	-
13. Mines	265.63	162.19	-	49.81
14. Basic metals	451.48	90.79	-	130.49
15. Metal products	1745.01	1370.88	-	504.36
16. Non-metals	170.46	47.94	-	36.47
17. Chemicals	405.86	288.60	-	131.48
18. Construction	722.13	680.83	-	85.37
19. Transport	1060.29	456.24	-	96.16

The optimum plan under such a situation is presented in Table 29. It would be seen from the table that all the sectors except fuel and power enter at their maximum levels. The fuel and power output would be only half of the targetted increase in the Third Plan, since it enters the final solution at its minimum level. The upper bound of fuel and power sector is not met by 53.84 crores of rupees. As the capital output ratio is 6.4 for the fuel and power sector, a capital of (53.84 x 6.4) 344.6 crores of rupees is needed in addition to the 10,000 crores of outlay estimated in the plan, if that sector is also to reach the maximum targetted.

Table 30 describes more or less the investment map the planner will have to follow for optimum allocations for varying levels of capital availability. The figures presented in the table are percentages and the total availability of capital is indicated at the top of the table. It can be seen that investment in the metal products (Sector 15) should be 19.75 percent when capital for investment is available at 6000 crores, 16.41 percent at 7218.58 crores and 23.70 percent at 10,000 crores of rupees. It is interesting to note that 38.07 percent of the investment has to be in the first five agricultural sectors at 6000 crores of capital availability and 35.04 percent when capital is available at 10,000 crores of rupees.

The above program could also be modified if the

Table 30. Optimum allocation of capital in the different sectors at varying levels of capital availability

Sectors	Capital allocation expressed in %			
	At 6000 crores	7218.58 crores	8775.12 crores	10,000 crores
1. Wheat and other cereals	12.04	11.13	9.16	8.04
2. Rice	6.35	10.55	8.68	7.62
3. Industrial crops	3.24	5.39	4.44	3.89
4. Livestock products	7.09	5.89	4.85	4.26
5. Other agricultural products	9.35	15.56	12.79	11.23
6. Food manufactures	1.14	0.95	1.57	1.37
7. Vegetable oils	0.58	0.48	0.79	0.69
8. Wood	2.00	1.66	2.97	2.61
9. Jute	0.43	0.36	0.59	0.52
10. Textiles	2.16	1.80	2.75	2.41
11. Leather and leather goods	1.85	1.54	2.53	2.23
12. Fuel and power	5.74	4.77	3.93	3.44
13. Mining	2.00	1.65	1.36	2.39
14. Basic metal industries	5.11	4.25	3.49	6.13
15. Metal products	19.75	16.41	21.48	23.70
16. Non-metal products	1.15	0.96	1.58	1.38
17. Chemicals	4.64	3.86	6.35	5.58
18. Construction	0.31	0.26	0.43	0.38
19. Transport	15.04	12.51	10.29	12.15
Total	99.97	99.98	100.03	100.03

objective of the planners is to maximize certain industrial sectors, which are considered to be the core of the plan. The weights in the maximizing function could be altered to suit the objectives of the planners and the same model could provide an optimum under the changed ideals. Unlike the aggregate models, this simple model provides sectoral requirements for the 19 sectors in the Indian economy. The model could be easily extended to the 1955-56 table containing 36 sectors of the economy. Additional resource restrictions will greatly enhance the applicability of the model to practical planning problems.

Primal and the dual

The general equilibrium where the value of outputs are maximized can be written as

$$\text{Maximize } p_1X_1 + p_2X_2 + \cdots + p_nX_n$$

subject to

$$\begin{array}{cccccccc} a_{11}X_1 & + & a_{12}X_2 & + \cdots + & a_{1n}X_n & \leq & r_1 \\ a_{21}X_1 & + & a_{22}X_2 & + \cdots + & a_{2n}X_n & \leq & r_2 \\ \vdots & & \vdots & & \vdots & & \vdots \\ a_{m1}X_1 & + & a_{m2}X_2 & + \cdots + & a_{mn}X_n & \leq & r_m \end{array} \qquad (5.20)$$

$$X_i \geq 0$$

where p_i is the price of output X_i and a_{ij}'s are input coefficients and r_i's activity levels.

In the maximizing problem, the value of total output (price x output) is maximized. The dual of the maximizing problem is minimizing costs, which can be written as

$$\text{Minimize } r_1V_1 + r_2V_2 + \dots + r_mV_m$$

subject to

$$\begin{aligned} a_{11}V_1 + a_{21}V_2 + \dots + a_{m1}V_m &\geq p_1 \\ a_{12}V_1 + a_{22}V_2 + \dots + a_{m2}V_m &\geq p_2 \\ \vdots \qquad \vdots \qquad\qquad \vdots \quad &\quad \vdots \\ a_{1n}V_1 + a_{2n}V_2 + \dots + a_{mn}V_m &\geq p_n \\ V_i \geq 0 \end{aligned} \tag{5.21}$$

In the dual or the minimizing case the minimizing function denotes that the total factor income should be the least. Thus, the dual problem is to find the values for the scarce resources such that the total value of resources or incomes is the minimum consistent with no activity having unimputed surplus. According to one fundamental theorem in linear programming, a linear program has a solution if both the primal and its dual are feasible. Therefore, if we can find an equilibrium or optimal solution for the outputs of the general equilibrium, then there is solution also for factor returns. Economic theory also tells us that the demand and supply functions are such as to force this equality. In the words of Dorfman, Samuelson and Solow (1958), there is hidden in every competitive general equilibrium system a maximum problem for value of output and a minimum problem for factor returns.

In our formulation of the maximizing problem in System 5.18 outputs are maximized. One can also formulate its dual in which the optimum will yield the equilibrium prices as can be seen from System 5.21. Time and funds prevented us from getting the solution for the dual. Since our purpose is to mainly provide a theoretical model, empirical verification is only secondary. However, the shadow prices obtained from the solution of the variable capital programming model are presented in Table 31.

It would be seen from the table that the shadow prices of the agricultural sectors, for which a weight of 100 was given, are around 9.9, while the industrial sectors, with a weight of 1, range from 0.026 in the case of mining to 0.097 in the case of fuel and power. One of the reasons for the fuel and power sector entering the solution at its minimum is probably the high imputed price for that sector. A more realistic approach might be to try the actual price indices for the different sectors with the input-output table

Table 31. Imputed values or shadow prices for the different sectors

Sectors	At 10,000 crores	At 8775.12 crores
1. Wheat and other cereals	9.9628	9.9485
2. Rice	9.9628	9.9485
3. Industrial crops	9.9628	9.9485
4. Livestock and products	9.9628	9.9485
5. Other agricultural products	9.9628	9.9485
6. Food manufactures	0.0785	0.0702
7. Vegetable oils	0.0861	0.0808
8. Wood	0.0415	0.0191
9. Jute	0.0745	0.0647
10. Textiles	0.0665	0.0536
11. Leather and leather goods	0.0415	0.0191
12. Fuel and power	-0.0969	-0.1723
13. Mining	0.0261	-0.0021
14. Basic metal industries	0.0277	-
15. Metal products	0.0277	-
16. Non-metal products	0.0415	0.0191
17. Chemicals	0.0347	0.0098
18. Construction	0.0932	0.0906
19. Transport	-	-0.0383
Capital	0.0308	0.0425

computed at producers' prices rather than work with a table computed at market prices, since the ratio between the prices of different sectors may not remain constant over time.

Before concluding, we would like to stress that the variable capital programming model has the same limitations described for the earlier model. Absence of data did not permit us to extend the scope of the model. In the face of all the deficiencies, the model does indicate to the planner the optimum capital allocations to the different sectors, assuming that agricultural production should be met at the end of the Third Plan.

SUMMARY

This study, besides briefly reviewing the nature of input-output work carried on in the different parts of the world, considers the important differences between the partial analysis and the input-output approach. The essential similarities and differences between the Leontief's input-output system and the general equilibrium are also considered.

The macro economic growth models, as the Harrod-Domar model and the Mahalanobis' Planning Model for India are discussed with a view to showing how the aggregate growth models have close conceptual similarity to the dynamic input-output system. The major departure between the aggregate models and the Leontief's model is that the global coefficients of the former models are disaggregated into sectoral coefficients in the latter system. We have also discussed at length the programming formulations of the Mahalanobis' model by Uma Datta (1961) and Komiya (1959) since the introduction of choice leads to optimal solutions and renders the Mahalanobis' model more operational, which is otherwise deterministic. The Short Term Planning Model for India developed by Padma Desai (1961) and the long term Demonstration Planning Model for India projected by Sandee within the input-output framework are considered to demonstrate

the usefulness of input-output data for building models to aid economic development.

In this study, we have discussed some of the problems inherent in the aggregation of sectors and have also considered the conceptual background for the inter-industry table published by the Indian Statistical Institute. Nature of data, sources and estimation procedures have also been touched upon.

In the empirical part of the study, a structural analysis of the Indian economy has been made using the nineteen sector input-output projection for 1960-61. The types of productive sectors of the Indian economy have been classified by type of input and by use of output after Chenery and Watanabe (1958) into (1) intermediate primary production, (2) intermediate manufacture, (3) final manufacture and (4) final primary production. Such a classification for the Indian economy is compared with those of Italy, Japan and the United States of America.

By using the inverse matrix $(I - A)^{-1}$, we have computed the output-composition needed to support the different final demand vectors, as exports, government and private consumption and investment. In the developing economies, efforts are directed towards replacing the imported goods with products of indigenous origin. The estimated changes in the output-composition to substitute imports have been indicated. The amount of primary and purchased inputs required by the different sectors and their implications in economic interdependence have also been discussed.

Another aspect of the empirical analysis is the programming formulation of the open Leontief system. In India during the Third Five Year Plan, a sum of ten thousand crores is estimated to be invested for the development of the agricultural and industrial sectors. The 19 sector table was aggregated to 10 sectors and output levels at the end of the Third Plan were estimated from the figures provided by the Planning Commission (1961). Capital was considered to be the scarce resource and capital-output ratios were built up for the 10 aggregated sectors. The programming problem was one of maximizing output or national income subject to resource and structural constraints. The simplex solution indicated that all the maximum outputs, except that of power and transport, could be met with the proposed investment of ten thousand crores of rupees.

Out of the total investment of ten thousand crores of rupees, more than fifty percent of the capital is anticipated from outside sources, as foreign investments, foreign aid, and loans. Assuming that the country is not in a position to secure the foreign capital as expected, then the planners are faced with the problem of allocating the available capital in an optimal manner. In a country like India, agricultural targets are essential and have to be met before one could think of increasing targets in the industrial sectors. So higher weights were assigned for the agricultural sectors and a variable capital programming was run to find the optimum output levels at different discrete levels of capital availability. The optimum output levels at 6000, 7218.58, 8775.12 and 10,000 crores of rupees are presented in Table 26 to 29.

ACKNOWLEDGMENTS

The author expresses deep appreciation to Dr. Earl O. Heady for his keen interest and guidance throughout the period of graduate study and in the preparation of this manuscript. The author considers it a rare privilege to have been associated with Dr. Heady, an international authority in agricultural economics, and is grateful for the inspiration he drew from Prof. Heady. Also, the author is greatly indebted to Professors William B. Murray, George M. Beal, Erik Thorbecke and Frank Riecken, members of the graduate committee, for their guidance and constructive suggestions during the course of the graduate study.

To Dr. Karl A. Fox and Dr. Gerhard Tintner, thanks are due from the author for the encouragement and help received from them.

The programming computations of this study would not have been possible but for the facility afforded by Mr. Henry Hibshman of the Esso Research and Engineering Co., New Jersey and the author heartily thanks him for the timely help. The author also appreciates the assistance rendered by Mr. Dale Grosvenor in the different stages of the computation.

Finally, the author expresses his gratitude to Dean Krishnamurthi for affording the opportunity for this graduate study and to the Government of Madras, the University of Tennessee and the Technical Cooperation Mission for the financial support.

LITERATURE CITED

Aitken, A. C. 1951. Determinants and matrices. 7th ed. Edinburg and London, Oliver and Boyd.

Anderson, R. L. and Bancroft, T. A. 1952. Statistical theory in research. New York, N. Y., McGraw-Hill Book Co., Inc.

Artle, Roland. 1959. Studies in the structure of Stockholm economy. Stockholm, Business Research Institute, Stockholm School of Economics.

Balderston, J. 1954. Models of general economic equilibrium. In Morgenstern, Oscar, ed. Economic activity analysis. pp. 3-41. New York, N. Y., John Wiley and Sons, Inc.

________ and Whitin, T. M. 1954. Aggregation in the input-output model. In Morgenstern, Oscar, ed. Economic activity analysis. pp. 79-144. New York, N. Y., John Wiley and Sons, Inc.

Barna, Tibor, ed. 1956. The structural interdependence of the economy. New York, N. Y., John Wiley and Sons, Inc.

Barnett, H. J. 1954. Specific industry output projections in National Bureau of Economic Research, long range economic projection. Princeton, N. J., Princeton University Press.

Barone, E. 1938. The ministry of production in the collectivist state. In Hayek, F. A., ed. Collectivist economic planning. pp. 254-290. London, George Routledge and Sons, Limited.

Bauchet, Pierre. 1955. Les tableaux economiquisi analyse de la region Lorraine. Paris, Genin.

Boles, J. N. 1956. Shortcuts in programming computations. J. Farm Econ. 4: 981-990.

Candler, Wilfred. 1957. Linear programming with stochastic yields. Unpublished Ph.D. thesis. Ames, Iowa, Library, Iowa State University of Science and Technology.

________. 1956. A modified simplex solution for linear programming with variable capital restrictions. J. Farm Econ. 4: 940-955.

Carter, Harold O. 1958. Regional input-output analysis of agriculture and industry. Unpublished Ph.D. thesis. Ames, Iowa, Library, Iowa State University of Science and Technology.

________ and Heady, Earl O. 1959. An input-output analysis emphasizing regional and commodity sectors of agriculture. Iowa Agr. Expt. Sta. Res. Bul. 469.

Cassel, Gustav. 1932. Theory of social economy. New York, N. Y., Harcourt Brace and Company, Inc.

Chakravarthi, A. K. 1961. On some aspects of the structure of the Indian economy. Paper presented at the International Input-Output Conference on Input-Output Techniques. Geneva.

Chakravarty, S. 1959. The logic of investment planning. Contributions to economic analysis XVIII. Amsterdam, North Holland Publishing Company.

Chenery, H. B. 1953. Regional analysis in the structure and growth of the Italian economy. (Published in Italian in L'Industria) 1:3-26.

________ and Clark, P. G. 1959. Interindustry economics. New York, N. Y., John Wiley and Sons, Inc.

________ and Watanabe, T. 1958. International comparisons of the structure of production. Econometrica 4: 487-521.

Christ, Carl. 1955. A review of input-output analysis. In Input-output analysis: an appraisal. Studies in income and wealth, Volume 18. A report of the National Bureau of Economic Research, New York. pp. 137-149. Princeton, New Jersey, Princeton University Press.

Clark, Colin. 1951. The conditions of economic progress. 2nd ed. London, Macmillan and Company.

Cornfield, J., Evans, W. D. and Hoffenberg. 1947. Full employment patterns, 1950. Monthly Labor Review 2:163-190 and 3:420-432.

Dantzig, G. B. 1949. Maximization of linear form whose variables are subject to linear inequalities. Headquarters U. S. Air Force. The comptroller. Washington, D. C. (Mimeographed).

Derwa, Leon. 1957. Une nouvelle methode d'analyse de la structure economique. Belgium, Revue du Consiel Economique Wallon.

Domar, Evsey. 1957. Essays in the theory of economic growth. New York, N. Y., Oxford University Press.

Dorfman, R., Samuelson, P. and Solow, R. M. 1958. Linear programming and economic analysis. New York, N. Y., McGraw-Hill Book Company, Inc.

Dresch, F. W. 1938. Index numbers and the general economic equilibrium. Bulletin of the American Mathematical Society 44: 134-141.

Fox, K. A. 1953. A spatial equilibrium model of the livestock-feed economy in the United States. Econometrica 4:547-566.

________ and Sengupta, J. K. 1961. Uses of the input-output and related techniques in partial analysis: the agricultural sector. Paper presented at the International Conference on Input-Output Techniques. Geneva (Mimeographed). U. N. Statistical Office, New York, N. Y.

Fisher, W. D. 1958. Criteria for aggregation in input-output analysis. Review of Economics and Statistics 3:250-260.

Goodwin, R. M. 1953. Static and dynamic linear general equilibrium models. Input-output relations. Proceedings of a conference on interindustrial relations held at Driebergen, Holland. pp. 54-98. ed. The Netherlands Economic Institute, H. E. Stewfert Kroese. Leiden.

________ and Sengupta, S. ca. 1953. Interindustry table for 1950-51. Calcutta, Indian Statistical Institute.

Haldane, J. B. S. 1955. The maximization of national income. Sankhya, The Indian Journal of Statistics 16: 1-2.

Harrod, R. F. 1948. Towards a dynamic economics. London, Macmillan and Co.

Hawkins, D. and Simon, H. A. 1949. Note: some conditions of macro economic stability. Econometrica 17: 245-248.

Heady, Earl O. 1962. Agricultural policy under economic development. Ames, Iowa, Iowa State University Press.

________. 1961. Uses and concepts in supply analysis. In Heady, E. O., Baker, C. B., Diesslin, H. G., Kehrberg and Staniforth, S., eds. Agricultural supply functions. pp. 3-28. Ames, Iowa, Iowa State University Press.

________ and Candler, W. 1958. Linear programming methods. Ames, Iowa, Iowa State University Press.

Henderson, J. M. 1958. The efficiency of the coal industry. Cambridge, Massachusetts, Harvard University Press.

Hicks, J. R. 1946. Value and capital. 2nd ed. London, Oxford University Press.

India. ca. 1960. Input-output projections for 1960-61. New Delhi, Institute of Public Opinion.

________. 1957. Interindustrial relations in the Indian Union. Paper presented by the Indian Statistical Institute at the Conference on Research in National Income. New Delhi.

________. 1952. The national sample survey--no. 2. Department of Economic Affairs, Ministry of Finance, Government of India.

________. 1956. Second five year plan. Planning Commission, Government of India. New Delhi.

________. 1961. Third five year plan. Planning Commission, Government of India. New Delhi.

Isard, W. 1951. Inter-regional and regional input-output analysis. The Review of Economics and Statistics 4: 318-328.

________. 1953. Some empirical results and problems of regional input-output analysis. In Leontief, W., ed. Studies in the structure of the American economy. pp. 116-184. New York, N. Y., Oxford University Press.

Japan. 1958. Inter-regional input-output table for the Kinki area and the rest of Japan. Kansai Economic Federation, Osaki.

Keynes, John Maynard. 1935. The general theory of employment. Employment, interest and money. New York, N. Y., Harcourt Brace and Company, Inc.

Klein, L. R. 1946. Macroeconomics and the theory of rational behavior. Econometrica 14: 93-108.

Komiya, Ryutaro. 1959. A note on Professor Mahalanobis' Model of Indian Economic Planning. The Review of Economics and Statistics 1:29-36.

Koopmans, T. 1957. Analysis of production as an efficient combination of activities. In Koopmans, T., ed. Activity analysis of production and allocations. pp. 33-97. New York, N. Y., John Wiley and Sons.

Leontief, W. 1956. Interregional theory. In Leontief, W., ed. Studies in the structure of the American economy. pp. 93-115. New York, N. Y., Oxford University Press.

________. 1949. Recent developments in the study of interindustrial relations. Papers and proceedings of the American Economic Association. p. 216. Evanston, Illinois.

________. 1951. The structure of the American economy, 1919-1939. 2nd ed. New York, N. Y., Oxford University Press.

________ and Members of the Harvard Economic Research Project. 1953. Studies in the structure of the American economy. New York, N. Y., Oxford University Press.

Mahalanobis, P. C. 1955. The approach of operational research. Sankhya, The Indian Journal of Statistics 16: 3-62.

________. 1953. Some observations on the process of growth of national income. Sankhya, The Indian Journal of Statistics 12: 307-312.

Marshall, Alfred. 1920. Principles of economics. 8th ed. London, Macmillan.

May, K. 1947. Technological change and aggregation. Econometrica 15: 303-313.

Moore, H. L. 1929. Synthetic economics. New York, N. Y. The Macmillan Company.

Moses, L. N. 1955. The stability of interregional trading patterns and input-output analysis. American Economic Review 5: 803-832.

Mukherjee, M. and Shastry, N. S. R. 1959. Estimate of the tangible wealth in India. Income and Wealth Series VIII. pp. 365-387. London, Bowes and Bowes.

Neisser, H. and Modigliani, F. 1953. National incomes and international trade. Urbana, Illinois, Illinois University Press.

Netherlands. 1954. Amsterdam Municipal Bureau of Statistics, Stedeligke jaarkeningen van Let Bureau van Statistick du Germeente, Amsterdam. Supplement 4.

Padma, Desai. 1961. A short-term planning model for the Indian economy. The Review of Economics and Statistics 2: 186-193.

Pareto, Vilfredo. 1911. Manuel d'Economic Politique. V. Girard et E. Briere, Paris, 1907 and Economie Mathematique, Encyclopedie des Sciences Mathematiques, Paris.

Patel, Surendra J. 1961. Observations on the Third Five Year Plan. Working paper, Indian Statistical Institute. Calcutta.

Peterson, G. A. 1953. Use of input-output analysis in estimating the interdependence of agriculture and other economic sectors. Unpublished Ph.D. thesis. Ames, Iowa, Library, Iowa State University of Science and Technology.

________ and Heady, Earl O. 1955. Application of input-output analysis to a simple model emphasizing agriculture. Iowa Agr. Expt. Sta. Res. Bul. 427.

Quesnay, Francois. 1894. Tableau economique. London, reproduced in facsimile for the British Economic Association. (First printed in 1758).

Samuelson, P. A. 1957. Abstract of a theorem concerning substitutability in open Leontief models. In Koopmans, T., ed. Activity analysis of production and allocation. pp. 142-146. New York, N. Y., John Wiley and Sons, Inc.

________. 1948. Foundations of economic analysis. Cambridge, Massachusetts, Harvard University Press.

Sandee, J. 1960. A demonstration planning model for India. Bombay, Asia Publishing House.

Schnittker, John Alvin. 1956. Application of input-output analysis to a regional model stressing agriculture. Unpublished Ph.D. thesis. Ames, Iowa, Library, Iowa State University of Science and Technology.

________ and Heady, Earl O. 1958. Application of input-output analysis to a regional model emphasizing agriculture. Iowa Agr. Expt. Sta. Res. Bul. 454.

Shou Shan Pu. 1946. A note on macroeconomics. Econometrica 14: 303-313.

Tinbergen, J. 1958. The design of development. Baltimore, Maryland, The Johns Hopkins Press.

Uma Datta. 1957. A preliminary study of interindustry relations in India. Working paper 7. India, Indian Statistical Institute, Calcutta.

________. 1961. Growth of the Indian economy during the third plan period. Paper presented at the Second Econometric Conference, Waltair, India.

United Nations. Economic Commission for Latin America. 1956. Analyses and projections of economic development. II. Economic development of Brazil. New York, N. Y.

________. Economic Commission for Latin America. 1957. Analyses and projections of economic development. III. Economic development of Columbia. New York, N. Y.

________. Economic Commission for Latin America. 1958. Analyses and projections of economic development. IV. Economic development of Argentina. New York, N. Y.

United States. Department of Labor. 1952. Table 1, Inter-industry flow of goods and services by industry of origin and destination, continental United States, 1947.

von Neumann and Morgenstern, O. 1944. Theory of games and economic behavior. Princeton, N. J., Princeton University Press.

Wald, Abraham. 1934. Uber die eindeutige positive Losbarkeit der neuen Produktionsgleichungen, Ergebrisse eines Mathematischen Kolloquiums.

Walras, Leon. 1954. Elements of pure economics. (W. Jaffe, tr.). (First published in French in 1874). Homewood, Illinois, Richard D. Irwin.

Waugh, V. 1950. Inversion of Leontief matrix by power series. Econometrica 12: 142-154.

APPENDIX A

Figure 3. Inter-industry relations of the Indian economy, 1953-54 (in crores of rupees)

Activities receiving from Products flowing to		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
primary	1. agriculture.	259.81	-	808.95	-	-	-	-	-	2.65	-	0.01	250.48	128.19	0.67	39.09	0.12
	2. plantations.	-	-	-	-	-	-	-	0.98	-	-	-	44.90	-	0.01	-	-
	3. animal husbandry.	832.74	5.33	0.95	0.36	0.11	0.05	0.05	2.33	1.74	0.02	2.65	0.47	0.39	6.74	0.07	0.01
	4. coke and coal.	-	-	0.07	3.32	-	6.40	0.45	0.83	2.51	1.40	0.06	1.95	2.53	0.25	0.48	1.25
	5. all other minings.	1.87	0.87	0.03	1.59	4.19	6.14	5.95	1.93	5.18	2.01	2.06	4.12	3.59	0.27	2.72	3.18
large scale manufacture	6. iron and steel.	-	-	0.14	-	0.21	22.11	1.11	27.23	0.39	0.25	1.33	0.25	0.72	-	0.53	0.35
	7. non-ferrous metals.	-	-	-	-	-	5.47	6.30	8.68	1.23	-	0.01	0.76	-	0.08	-	0.07
	8. engineering.	3.07	0.84	0.96	1.94	1.89	-	-	5.40	0.41	-	0.01	3.19	-	-	-	-
	9. chemicals.	7.68	2.29	1.78	1.00	0.08	0.66	0.20	2.43	35.02	0.04	0.92	1.47	10.44	1.88	0.73	1.32
	10. cement.	-	-	-	-	-	-	-	0.12	-	0.63	0.05	-	-	-	-	-
	11. other building materials.	-	-	-	-	-	0.11	-	0.31	1.19	0.07	0.25	4.42	0.14	0.04	-	0.23
	12. food, drink, tobacco etc.	6.10	0.08	28.89	0.08	-	-	-	0.06	8.32	-	-	42.99	0.03	-	1.46	-
	13. cotton textile.	-	-	-	-	-	-	-	0.40	-	-	0.02	0.04	113.97	3.74	0.60	-
	14. other textile.	-	-	-	-	-	-	-	0.24	-	-	-	-	5.98	10.97	1.93	-
	15. jute and other fibre.	-	-	0.01	0.03	0.85	0.07	0.03	0.04	2.39	2.86	0.01	7.91	2.83	0.07	5.27	0.24
	16. glass and ceramics.	-	-	-	-	-	-	-	0.26	0.38	-	-	0.38	-	-	-	0.20
	17. leather and rubber.	0.20	0.07	-	-	-	-	-	1.54	-	-	0.08	-	-	-	0.07	-
	18. paper, printing & stationery.	-	-	-	0.09	-	0.08	-	0.13	0.93	0.62	0.74	2.30	0.86	0.39	0.17	0.08
	19. electricity.	-	-	0.01	1.22	0.46	1.23	0.08	0.96	1.09	0.70	0.22	1.63	6.82	0.39	1.20	0.33
small scale manuf.	20. metal ware & metal working.	14.05	-	1.75	-	-	-	-	-	-	-	-	-	-	-	-	-
	21. building mat. & wood manf.	22.95	0.20	2.18	-	1.01	-	-	-	-	-	-	-	-	-	-	-
	22. textile and textile products.	0.04	-	0.18	-	-	-	-	-	-	-	-	-	-	-	-	-
	23. food, drink, tobacco etc.	6.00	0.09	20.23	-	-	-	-	-	-	-	-	0.17	-	-	-	-
	24. glass and ceramics.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	25. leather and leather products.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	26. other products.	32.00	-	-	-	-	-	-	-	-	-	-	-	-	-	-	0.01
tertiary activities	27. railway & communication.	0.38	0.03	1.00	0.10	0.06	2.56	0.99	3.25	3.65	1.82	0.48	10.58	6.69	0.55	2.59	0.01
	28. other transport.	1.35	0.15	0.97	0.15	0.03	0.32	0.20	0.43	3.80	0.58	0.03	6.82	4.73	1.48	2.64	0.07
	29. trade & distribution.	7.82	0.76	9.07	1.14	0.41	5.09	5.16	7.41	21.67	4.36	1.89	51.23	40.42	9.18	21.41	0.39
	30. bank, insurance & co-operative.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	31. professions.	3.79	-	54.63	-	-	-	-	-	3.12	-	-	-	-	-	-	-
	32. constructions.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	33. residential property.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	34. public administration.	-	-	-	0.03	0.04	0.61	0.05	0.22	0.24	0.02	0.90	1.28	1.05	0.09	0.02	0.11
	35. defence materials (incl. empl.)	-	-	-	0.57	0.85	-	-	-	-	-	-	-	-	-	-	-
	36. unclassified: large scale.	-	-	0.41	0.68	0.33	0.08	0.08	1.13	2.20	0.39	0.83	0.32	0.58	0.66	0.13	0.35
	total material input.	1199.85	10.71	932.21	12.30	10.52	51.28	20.63	66.31	98.11	15.77	12.55	437.66	329.96	36.86	81.11	8.32
	commodity taxes.	21.41	-	3.93	-	53.02	3.93	3.47	30.83	26.08	1.37	9.62	92.88	50.07	8.44	11.34	1.99
	gross domestic prod. at factor cost.	4034.95	46.58	1205.45	57.65	50.21	54.43	5.35	57.90	61.37	11.32	12.98	136.03	169.31	17.61	48.83	8.02
	output with tax.	5256.21	57.29	2141.59	69.95	113.75	109.64	29.45	155.04	85.56	28.46	35.15	66.57	549.34	62.91	141.28	18.33
	wage income.	239.09	38.95	335.90	21.70	9.92	21.99	2.70	31.26	22.46	3.40	7.27	33.75	112.58	9.57	33.25	4.94
	non-wage income.	3795.86	7.63	869.55	35.95	40.29	32.44	2.60	26.64	38.91	7.92	5.71	102.28	56.73	8.04	15.58	3.08

15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	SUB-TOTAL	
39.09	0.12	-	0.11	-	-	0.09	-	141.84	0.05	-	0.45	4.08	-	53.42	-	-	3.90	-		2.84	-	1698.75	
-	-	5.49	-	-	-	-	-	-	-	-	0.21	-	-	3.76	-	-	-	-		-	-	55.35	
0.07	0.01	3.37	1.61	-	1.62	5.76	3.02	184.61	0.88	6.47	14.23	2.12	97.52	33.05	-	0.32	2.15	-		-	0.01	1210.13	
0.48	1.25	0.55	0.90	7.29	0.38	0.67	0.13	0.44	0.09	-	1.17	17.74	1.10	0.97	-	0.99	1.46	-		0.37	0.01	55.76	
2.72	3.18	0.57	0.43	1.46	0.56	4.85	0.67	1.59	0.46	0.36	1.96	2.35	38.37	2.87	0.04	4.51	19.93	0.50		17.68	0.34	145.15	
0.53	0.35	0.21	0.06	-	7.73	0.21	-	-	-	0.04	0.11	7.56	0.72	0.70	-	-	44.89	3.23		2.23	0.01	122.51	
-	0.07	-	0.04	0.18	10.71	-	-	-	-	-	3.29	0.36	-	-	-	3.30	0.01	-		1.64	1.12	40.07	
-	-	0.01	-	3.74	-	0.01	-	0.02	-	-	3.29	20.74	32.57	0.92	-	8.81	30.35	0.43		16.33	-	134.93	
0.73	1.32	1.89	3.02	-	0.19	0.39	1.36	3.68	0.10	0.41	80.46	1.08	2.71	0.65	-	14.99	13.80	-		2.71	0.22	192.60	
-	-	-	-	-	-	-	-	-	-	-	-	0.54	-	0.85	-	-	34.36	2.32		-	-	38.87	
-	0.23	0.04	0.06	-	-	0.45	-	0.04	-	-	-	2.13	-	0.76	-	3.13	17.82	2.97		-	-	34.16	
1.46	-	0.83	0.24	-	-	-	0.17	35.30	0.01	0.01	0.51	0.96	1.35	38.94	-	-	-	-		-	-	166.33	
0.60	-	4.18	0.23	-	-	-	40.27	-	-	1.33	0.13	0.54	-	-	-	2.42	-	-		-	0.84	168.71	
1.93	-	0.01	-	-	-	-	11.75	-	-	-	-	-	0.75	-	-	-	-	-		-	-	31.63	
5.27	0.24	0.30	0.39	-	-	-	0.63	0.17	-	-	0.06	0.04	0.34	4.18	-	-	-	-		-	-	28.72	
-	0.20	-	-	0.03	-	-	-	-	-	-	-	-	-	-	-	6.68	3.69	0.81		-	-	12.43	
0.07	-	1.51	0.05	0.29	-	-	-	-	-	7.21	3.58	0.25	10.85	-	-	-	-	-		-	-	25.70	
0.17	0.08	0.60	6.87	-	0.09	0.39	0.49	0.63	0.02	0.01	1.07	0.97	0.55	1.65	1.93	4.70	1.34	-		1.96	0.01	29.67	
1.20	0.33	0.31	0.75	2.75	-	-	0.13	0.21	-	-	0.17	0.92	1.96	8.79	0.16	2.76	-	-		-	0.01	35.26	
-	-	-	-	-	-		-	-	-	0.01	-	-	1.21	0.31	-	11.98	5.71	-		-	-	35.02	
-	-	-	-	-	-	-	-	-	-	-	-	-	1.57	0.59	-	5.03	48.74	1.56		-	-	83.83	
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	5.15	-	-		-	-	5.37	
-	-	-	-	-	-	-	0.89	5.91	-	-	1.37	-	0.25	-	-	-	-	-		-	-	34.93	
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		-	-	-	
-	-	-	-	-	-	-	-	-	..	-	-	-	-	-	-	-	-	-		-	-	-	
-	0.01	-	-	-	-	6.50	0.14	7.71	-	9.00	16.97	-	-	0.91	-	10.51	46.25	-		-	-	130.00	
2.59	0.01	0.14	1.77	0.23	1.81	0.47	1.10	4.02	0.1[illegible]	0.64	5.14	2.52	8.12	22.21	2.64	5.71	16.79	0.67		3.60	0.05	112.50	
2.64	0.07	1.11	0.64	0.05	0.35	-	0.14	1.70	0.11	1.05	5.08	1.50	0.83	6.70	0.03	1.84	6.91	0.58		0.71	0.07	53.15	
21.41	0.39	5.65	8.68	1.25	5.71	2.15	5.62	19.45	1.09	7.62	44.96	6.41	35.90	47.03	0.14	17.95	69.52	4.54		17.19	0.57	488.84	
-	-	-	-	-	-	-	-	-	-	-	-	-	2.00	63.90	0.66	-	-	-		-	-	66.56	
-	-	-	-	-	-	-	0.33	0.44	-	-	-	-	7.59	6.64	10.16	-	44.25	3.64		-	-	134.61	
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		-	-	-	
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		-	-	-	
0.02	0.11	0.10	0.08	-	-	-	-	-	-	-	-	-	-	0.26	-	-	-	-		-	-	9.10	
-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-		2.75	-	4.17	
0.13	0.35	0.17	1.58	-	0.06	-	0.03	-	0.28	0.02	1.32	0.12	0.48	-	0.67	1.64	0.85	-		-	0.21	15.60	
81.11	8.32	27.04	27.51	17.27	29.21	21.94	66.87	406.78	3.22	34.18	182.24	72.93	246.74	300.01	16.43	112.42	412.72	21.2[illegible]		70.01	3.47	5396.41	
11.34	1.99	6.66	6.40	4.76	-	-	-	26.99	-	-	0.02	-	21.04	36.60	-	2.41	-	28.34		-	17.81	449.41	
48.83	8.02	13.57	33.57	29.57	26.23	80.67	97.03	219.18	11.52	50.84	131.26	244.03	116.09	1815.44	82.30	667.63	354.64	556.07	558.87	-	1.24	11067.74	
141.28	18.33	47.27	67.48	51.60	55.44	102.61	16[illegible].90	652.95	14.74	85.02	313.52	316.96	383.87	2152.05	98.73	782.46	767.36	605.68	558.87	70.01	22.52	16933.56	11537.15
33.25	4.94	5.94	19.25	9.89	4.99	11.29	11.47	32.65	0.62	3.40	23.51	167.22	38.17	149.90	39.99	510.74	354.64	8.73	558.87	-	0.34	2830.34	
15.58	3.08	7.63	14.32	19.68	21.24	69.38	85.56	186.53	10.90	47.44	107.75	76.81	77.92	1665.54	42.31	156.89	-	547.34	-	-	0.90	8237.40	

	28	29	30	31	32	33	34	35	36	SUB-TOTAL	house-hold con-sumption	public authorities (current)	govt. capital accounts	other capital accounts	exports (ex-factory)	imports (c.i.f.) (-)	net indirect taxes	output at market price
8	-	53.42	-	-	3.90	-		2.84	-	1698.75	3565.73	19.06	-	-	39.70	-70.20	3.17	5256.21
	-	3.76	-	-	-	-		-	-	55.35	2.68	-	-	-	0.02	- 0.09	- 0.67	57.29
2	97.52	33.05	-	0.32	2.15	-		-	0.01	1210.13	904.14	1.26	0.02	8.83	43.51	-15.69	-10.61	2141.59
4	1.10	0.97	-	0.99	1.46	-		0.37	0.01	55.76	12.28	0.42	-	-	2.95	- 0.72	- 0.64	69.95
5	38.37	2.87	0.04	4.51	19.93	0.50		17.68	0.34	145.15	28.95	0.61	-	-	21.41	-89.08	6.71	113.75
6	0.72	0.70	-	-	44.89	3.23		2.23	0.01	122.51	-	-	0.91	15.88	4.07	-23.46	-10.27	109.64
6	-	-	-	3.30	0.01	-		1.64	1.12	40.07	-	-	0.01	5.13	1.52	-14.42	- 2.86	29.45
4	32.57	0.92	-	8.81	30.35	0.43		16.33	-	134.93	20.63	-	27.33	129.67	3.09	-131.84	-28.77	155.04
8	2.71	0.65	-	14.99	13.80	-		2.71	0.22	192.60	71.93	1.19	0.04	-	5.46	-52.62	-33.04	185.56
4	-	0.85	-	-	34.36	2.32		-	-	38.87	-	-	-	-	0.59	- 0.75	-10.25	28.46
3	-	0.76	-	3.13	17.82	2.97		-	-	34.16	24.00	-	0.10	-	0.77	- 1.16	-22.72	35.15
6	1.35	38.94	-	-	-	-		-	-	166.33	448.86	-	-	-	103.20	-29.70	-22.12	666.57
4	-	-	-	2.42	-	-		-	0.84	168.71	377.02	-	0.03	-	74.54	-56.05	-14.91	549.36
	0.75	-	-	-	-	-		-	-	31.63	49.58	2.03	-	-	4.30	-23.73	- 0.90	62.91
4	0.34	4.18	-	-	-	-		-	-	28.72	1.24	-	-	-	112.63	- 0.26	- 1.05	141.28
	-	-	-	6.68	3.69	0.81		-	-	12.43	7.64	-	-	-	0.20	- 1.74	- 0.20	18.33
5	10.85	-	-	-	-	-		-	-	25.70	9.62	-	0.03	-	3.77	- 1.85	-	47.27
7	0.55	1.65	1.93	4.70	1.34	-		1.96	0.01	29.67	58.40	1.97	-	-	4.97	-18.57	- 8.96	67.48
2	1.96	8.79	0.16	2.76	-	-		-	0.01	35.26	12.59	3.75	-	-	-	-	-	51.60
	1.21	0.31	-	11.98	5.71	-		-	-	35.02	11.01	-	-	11.26	-	- 0.05	- 1.80	55.44
	1.57	0.59	-	5.03	48.74	1.56		-	-	83.83	6.70	-	-	28.80	-	-	-16.72	102.81
	-	-	-	5.15	-	-		-	-	5.37	142.30	-	-	-	9.77	-	6.46	163.90
	0.25	-	-	-	-	-		-	-	34.93	617.37	-	-	-	0.65	-	-	652.95
	-	-	-	-	-	-		-	-	-	14.74	-	-	-	-	-	-	14.74
	-	-	-	-	-	-		-	-	-	82.02	-	-	-	-	-	3.00	85.02
	-	0.91	-	10.51	46.25	-		-	-	130.00	164.41	-	-	13.00	6.34	- 0.23	-	313.52
2	8.12	22.21	2.64	5.71	16.79	0.67		3.60	0.05	112.50	175.10	17.00	0.67	5.97	5.72	-	-	316.96
0	0.83	6.70	0.03	1.84	6.91	0.58		0.71	0.07	53.15	317.64	0.68	0.73	4.53	7.14	-	-	383.87
1	35.90	47.03	0.14	17.95	69.52	4.54		17.19	0.57	488.84	1552.89	4.63	6.27	41.63	57.79	-	-	2152.05
	2.00	63.90	0.66	-	-	-		-	-	66.56	32.17	-	-	-	-	-	-	98.73
	7.59	6.64	10.16	-	44.25	3.64		-	-	134.61	588.70	59.15	-	-	-	-	-	782.46
	-	-	-	-	-	-		-	-	-	-	69.44	246.34	451.58	-	-	-	767.36
	-	-	-	-	-	-		-	-	-	599.82	5.86	-	-	-	-	-	605.68
	-	0.26	-	-	-	-		-	-	5.10	-	553.77	-	-	-	-	-	558.87
	-	-	-	-	-	-		2.75	-	4.17	-	67.82	-	-	-	- 1.98	-	70.01
2	0.48	-	0.67	1.64	0.85	-		-	0.21	15.60	-	-	0.52	8.95	1.61	- 3.24	- 0.92	22.52
3	246.74	300.01	16.43	112.42	412.72	21.2[illegible]		70.01	3.47	5396.41	9910.06	808.64	283.00	725.23	315.72	-537.43	-168.07	16933.56
	21.04	36.60	-	2.41	-	28.34		-	17.81	469.41								
3	116.09	1815.44	82.30	667.63	354.64	556.07	558.87	-	1.24	11067.74								
6	383.87	2152.05	98.73	782.46	767.36	605.68	558.87	70.01	22.52	16933.56	11537.15							11537.15
2	38.17	149.90	39.99	510.74	354.84	8.73	558.87	-	0.34	2830.34								
1	77.92	1665.54	42.31	156.89	-	547.34	-	-	0.90	8237.40								

APPENDIX B

Table 32. Input-output projections for India - 1960-61 (in crores of rupees at 19

Sales by \ Purchase by	1	2	3	4	5	6	7	
1. Wheat and other cereals	124.98	-	-	-	-	26.55	-	
2. Rice	5.64	83.97	2.02	25.91	3.46	-	-	
3. Industrial crops	-	3.02	63.81	-	-	191.91	307.55	
4. Livestock and products	44.10	44.10	12.70	97.81	55.70	64.69	-	
5. Other agricultural products	115.78	60.41	-	537.49	216.48	18.98	33.92	
6. Food manufactures	-	-	-	-	-	37.15	-	
7. Vegetable oils	5.22	2.82	-	24.45	3.17	-	-	
8. Wood	2.30	2.21	-	-	15.01	0.55	0.15	:
9. Jute	16.93	-15.30	8.77	-	6.93	6.78	1.34	
10. Textiles	-	-	-	-	-	-	-	
11. Leather and leather goods	-	-	-	-	-	-	-	
12. Fuel and power	0.42	0.81	-	-	6.93	12.63	2.98	
13. Mining	-	-	-	-	-	-	-	
14. Basic metal industries	-	-	-	-	-	-	-	
15. Metal products	-	0.20	-	-	5.77	4.37	8.53	
16. Non metal products	-	-	-	-	-	3.70	-	
17. Chemicals	1.04	1.01	-	-	5.77	2.71	1.18	
18. Construction	-	0.20	-	-	-	2.34	3.80	
19. Transport	33.86	18.93	43.25	16.03	62.34	21.01	7.86	1
20. Import	142.53	51.75	118.75	4.05	21.65	15.22	4.68	:
21. Govt. ordinary revenue	20.27	20.34	36.09	5.67	133.93	62.53	6.32	1
22. Trade and services	1026.14	468.96	8.27	129.27	615.37	77.27	39.41	
23. Wages	675.87	1323.93	778.11	876.27	1953.78	105.10	96.09	1(
Total	2215.08	2097.96	1071.77	1717.23	3106.29	653.29	513.81	2(

t 1952-53 prices)

	8	9	10	11	12	13	14	15	16	17	18	19
	-	-	2.31	-	-	-	-	-	-	-	-	-
	-	-	-	-	82.63	-	-	-	-	-	-	5.74
55	-	119.54	244.21	-	-	-	-	-	-	-	-	-
	0.27	-	22.74	41.88	124.43	-	-	-	-	-	-	-
92	-	-	-	-	-	-	-	-	0.55	-	-	118.83
	-	-	1.25	-	0.78	-	-	-	-	0.46	-	-
	-	-	0.39	1.29	-	-	-	-	-	15.93	-	5.41
15	20.85	0.97	6.55	5.27	26.47	--	0.15	1.45	0.24	7.09	42.87	13.19
34	-	0.54	3.28	-	-	-	0.24	0.07	8.04	0.30	-	-
	-	0.37	122.30	-	-	-	-	-	-	-	-	0.26
	0.48	0.25	1.16	5.27	-	-	-	-	-	-	-	1.06
98	2.10	3.32	14.55	13.98	11.14	3.04	21.83	4.14	12.87	14.14	30.71	28.67
	-	-	-	-	-	2.77	27.26	-	8.44	2.43	9.85	-
	-	-	-	-	0.49	-	0.26	137.46	-	1.18	15.68	6.40
53	1.94	4.77	15.03	-	13.77	2.38	5.36	42.26	11.20	6.03	31.20	73.41
	-	-	-	-	-	-	-	1.93	-	1.16	83.87	-
18	6.99	2.19	10.22	4.61	-	-	1.10	6.76	3.10	10.66	8.75	1.25
30	0.25	2.13	2.12	-	3.71	-	1.32	-	6.73	0.92	-	3.89
36	19.07	7.30	14.75	3.44	63.39	28.55	25.83	19.12	19.12	6.96	-	3.43
58	25.58	0.02	31.42	3.01	102.46	1.58	35.34	395.85	11.89	29.15	-	-
32	10.13	24.80	76.52	17.63	92.21	9.31	15.15	70.87	1.03	17.04	5.06	59.63
41	6.11	3.59	141.09	10.68	137.92	45.15	-	41.23	0.11	5.19	92.13	-
09	106.31	37.14	375.76	55.07	328.58	71.05	52.99	10.49	14.75	22.77	230.26	363.62
31	200.08	206.93	1085.65	162.13	987.98	163.83	186.83	731.67	98.07	141.41	550.38	684.78

Table 32. (Continued)

Purchase by / Sales by	Sum of intermediate demand 1-19	Exports 20	Govt. ordinary expenditure 21	Inve
1. Wheat and other cereals	153.84	-	-	
2. Rice	209.37	-	-	
3. Industrial crops	930.04	31.00	-	
4. Livestock and products	508.42	14.55	-	
5. Other agricultural products	1102.44	240.00	1.00	
6. Food manufactures	39.64	22.00	-	
7. Vegetable oils	58.68	33.00	-	
8. Wood	145.32	6.10	10.00	4
9. Jute	68.52	135.65	2.50	
10. Textiles	122.93	140.00	13.10	
11. Leather and leather goods	8.22	34.00	1.20	2
12. Fuel and power	184.26	14.00	18.00	
13. Mining	50.75	53.50	10.00	0
14. Basic metal industries	161.47	16.00	-	1
15. Metal products	226.21	23.00	17.00	312
16. Non metal products	90.66	5.00	-	2
17. Chemicals	67.34	5.00	-	
18. Construction	27.41	-	24.00	500
19. Transport	414.29	-	12.00	
20. Import	994.93	9.00	-	
21. Govt. ordinary revenue	684.53	-	-	
22. Trade and services	2847.97	-	-	
23. Wages	7477.93	45.12	829.17	275
Total	16575.17	826.92	937.97	1097

t. ordinary enditure 21	Investments 22	Consumers 23	Total final demand 20-23	Total goods supply national output 1-23	Tota
-	-	2061.00	2061.00	2214.84	
-	-	1889.24	1889.24	2098.61	
-	-	111.24	142.24	1072.28	
	-				
•	-	1235.94	1250.49	1758.91	
1.00	-	1751.66	1992.66	3095.10	
-	-	591.84	613.84	653.48	
-	-	422.62	455.62	514.30	
10.00	4.00	41.46	61.56	206.88	
2.50	-	1.50	139.65	208.17	
13.10	-	810.26	963.26	1086.29	
1.20	2.00	117.22	154.42	162.64	
18.00	-	780.19	812.19	996.45	
10.00	0.30	51.47	115.26	166.02	
-	1.80	11.23	29.03	190.50	
7.00	312.25	157.83	510.08	736.29	
-	2.00	0.87	7.87	98.53	
-	-	70.57	75.57	142.91	
4.00	500.00	-	524.00	551.40	
2.00	-	260.00	272.00	686.29	
-	-	39.24	48.24	1043.17	
-	-	545.19	545.19	1229.72	
-	-	-	-	2847.97	
9.17	275.18	1596.98	2746.45	10224.38	
7.97	1097.53	12547.55	15409.97	31985.14	

Total final demand 20-23	Total goods supply national output 1-23	Total net supply
2061.00	2214.84	2089.89
1889.24	2098.61	2013.58
142.24	1072.28	1008.05
1250.49	1758.91	1619.43
1992.66	3095.10	2886.20
613.84	653.48	616.09
455.62	514.30	513.87
61.56	206.88	179.25
139.65	208.17	206.35
963.26	1086.29	963.73
154.42	162.64	156.86
812.19	996.45	976.76
115.26	166.02	161.07
29.03	190.50	186.60
510.08	736.29	689.39
7.87	98.53	98.17
75.57	142.91	130.77
524.00	551.40	550.34
272.00	686.29	659.44
48.24	1043.17	-
545.19	1229.72	-
-	2847.97	-
2746.45	10224.38	-
15409.97	31985.14	-

APPENDIX C

Table 33. Inverse of the input-output projections for India, 1960-61

	1[a]	2	3	4	5
1[a]	1.05980	.00000	.00000	.00000	.00000
2	.00358	1.04233	.00292	.01715	.00222
3	.00714	.00760	1.06906	.01050	.00273
4	.02409	.02428	.01405	1.06644	.02135
5	.07138	.04253	.01308	.35545	1.08685
6	.00000	.00000	.00000	.00000	.00000
7	.00316	.00196	.00060	.01546	.00188
8	.00219	.00185	.00121	.00248	.00668
9	.00838	.00778	.00882	.00105	.00249
10	.00002	.00002	.00003	.00000	.00001
11	.00004	.00003	.00008	.00003	.00005
12	.00157	.00132	.00235	.00195	.00405
13	.00012	.00009	.00023	.00014	.00023
14	.00070	.00046	.00145	.00079	.00123
15	.00270	.00184	.00547	.00323	.00524
16	.00005	.00005	.00007	.00005	.00006
17	.00094	.00084	.00033	.00094	.00258
18	.00024	.00027	.00037	.00025	.00022
19	.01916	.01158	.04475	.01844	.02381

[a]Sectors 1 to 19 are the same as described in Table 32.

Table 33 (Continued)

	6	7	8	9	10
1	.04565	.00000	.00000	.00000	.00260
2	.00550	.00279	.00296	.00394	.00298
3	.34217	.64149	.00381	.61729	.27498
4	.12164	.01103	.00577	.01157	.03192
5	.08672	.08333	.02285	.01648	.01686
6	1.06031	.00001	.00015	.00006	.00143
7	.00298	1.00098	.00562	.00213	.00251
8	.00439	.00304	1.11787	.00921	.00994
9	.01500	.00820	.00028	1.00793	.00584
10	.00005	.00003	.00004	.00204	1.12689
11	.00012	.00009	.00285	.00138	.00132
12	.02598	.00991	.02178	.02259	.01913
13	.00134	.00107	.00176	.00167	.00097
14	.00382	.00524	.00639	.00746	.00454
15	.01654	.02437	.02637	.03428	.02168
16	.00684	.00129	.00078	.00187	.00057
17	.00576	.00340	.04155	.01266	.01234
18	.00493	.00793	.00246	.01106	.00269
19	.05720	.04636	.10986	.06660	.03176

Table 33 (Continued)

	11	12	13	14	15
1	.00000	.00003	.00000	.00000	.00000
2	.01337	.09062	.00412	.01308	.00367
3	.01095	.00281	.00118	.00273	.00175
4	.29711	.13781	.00459	.01834	.00502
5	.10672	.06199	.03553	.03947	.01410
6	.00019	.00085	.00002	.00013	.00007
7	.01653	.00289	.00165	.00258	.00199
8	.04352	.03259	.00517	.01005	.00593
9	.00058	.00091	.00016	.00164	.00072
10	.00001	.00003	.00007	.00007	.00002
11	1.03365	.00019	.00030	.00030	.00001
12	.09619	1.01636	.02813	.12975	.03479
13	.00098	.00003	1.01829	.14796	.02994
14	.00235	.00588	.00868	1.01381	.20205
15	.00865	.02439	.03705	.05576	1.07653
16	.00045	.00075	.00029	.00153	.00326
17	.03373	.00191	.00108	.00791	.01248
18	.00103	.00438	.00124	.00871	.00223
19	.04034	.07342	.18142	.17640	.06539

Table 33 (Continued)

	16	17	18	19
1	.00001	.00016	.00000	.00000
2	.01654	.01150	.00863	.01364
3	.05556	.08138	.01039	.00635
4	.02366	.01819	.01290	.01119
5	.06029	.03082	.01744	.19540
6	.00026	.00380	.00016	.00005
7	.00678	.12190	.00378	.00901
8	.02236	.06647	.09389	.02562
9	.08365	.00435	.01295	.00078
10	.00027	.00004	.00005	.00043
11	.00054	.00029	.00034	.00169
12	.16064	.11912	.09078	.04973
13	.09528	.02315	.03914	.00500
14	.03527	.02267	.04696	.03216
15	.16320	.06574	.09243	.11956
16	1.01178	.01049	.15439	.00133
17	.03989	1.08496	.02757	.00498
18	.07252	.00981	1.01201	.00644
19	.24408	.08409	.06304	1.02410